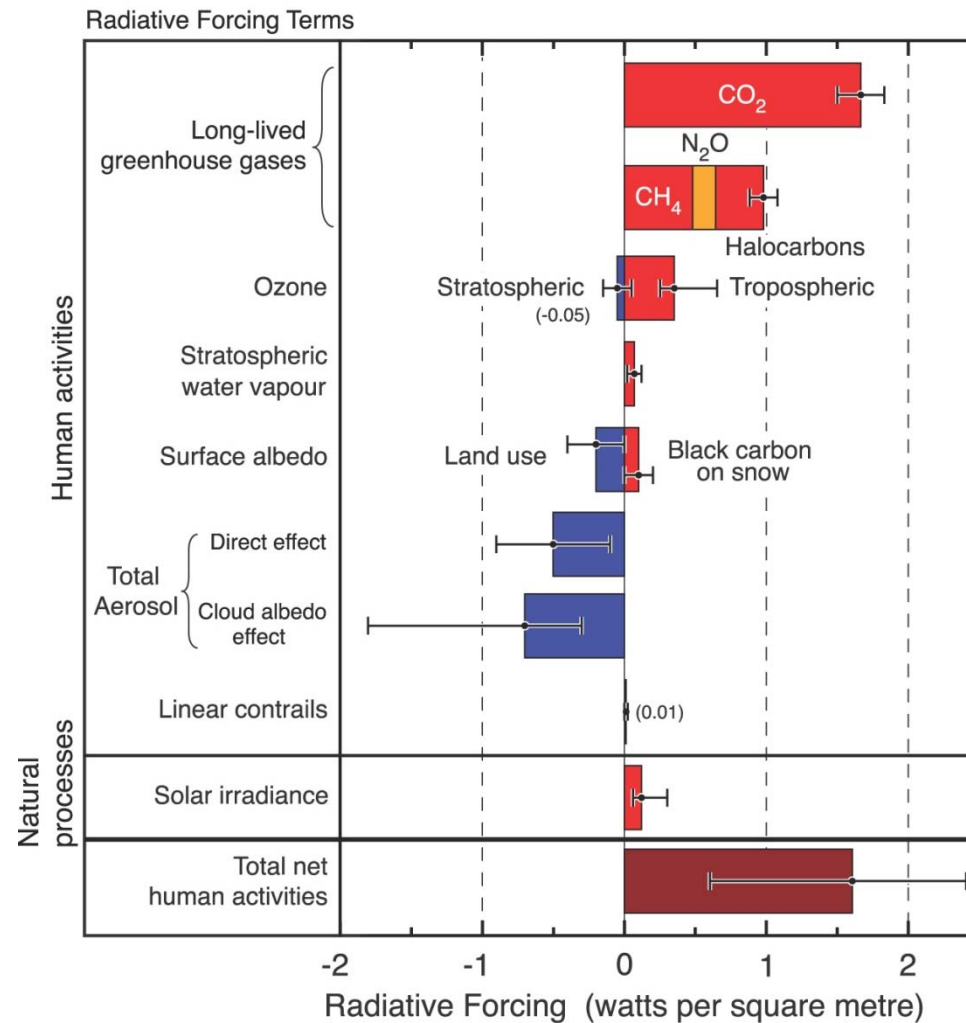


Lecture

Greenhouse gases: lifetimes and  
global warming potentials

# Contributions to Radiative Forcing of Climate (1750 to 2005)



# Controls on buildup of non-CO<sub>2</sub> greenhouse gases

$$M_{air} \cdot \frac{dX}{dt} = S(t) - k \cdot M_{air} \cdot X$$

$M_{air}$	total moles of air in the atmosphere (moles)
$X$	mole fraction of greenhouse gas, e.g. CH <sub>4</sub> , in atmosphere
$S(t)$	source of greenhouse gas in (moles/yr)
$k$	sink coefficient (1/years)
$1/k$	<b>atmospheric lifetime</b> (years)

Simple case: If  $S$  varies slowly enough (on a times scale much longer than  $1/k$  years) can assume **quasi-steady state**:

$$X(t) = S(t) / (k \cdot M_{air})$$

The atmospheric mole fraction is then proportional to the source strength, e.g. doubling of emissions leads to doubling of atmospheric amount, assuming  $k$  is constant.

# Controls on Methane (CH<sub>4</sub>) Increase

## Main Sources

Bogs & swamps (natural)  
Ruminants (cows, sheep, etc.)  
Rice Agriculture  
Coal, Oil, Gas production  
Land fills  
Biomass Burning

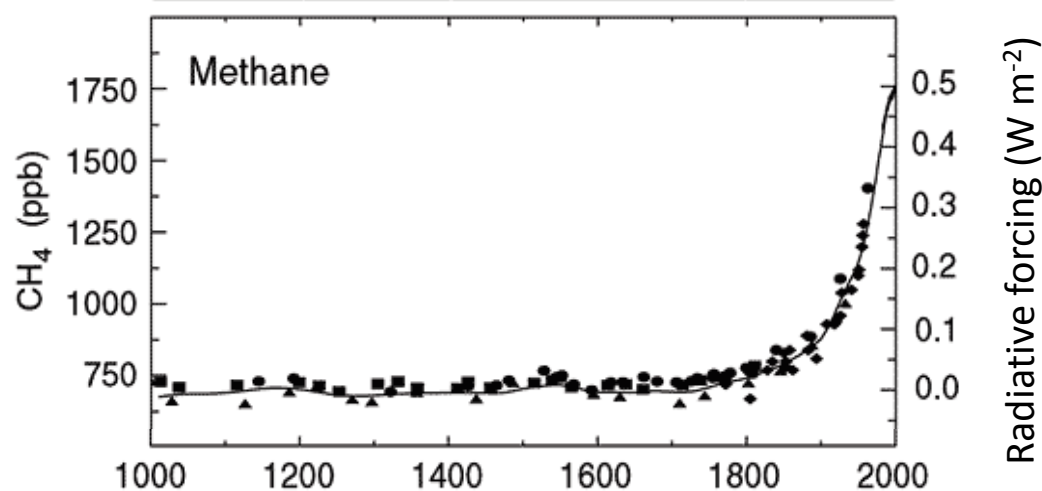
## Main Sink

Oxidation by OH radical

## Atmospheric Lifetime

~9 years

- Well-approximated by quasi-steady state relation
- Rise has largely stalled since ~1990



# Controls on Nitrous Oxide (N<sub>2</sub>O) Increase

## Main Sources

Soils (enhanced by expanding agriculture & N-fertilizer usage, produced microbially)

Oceans (natural)

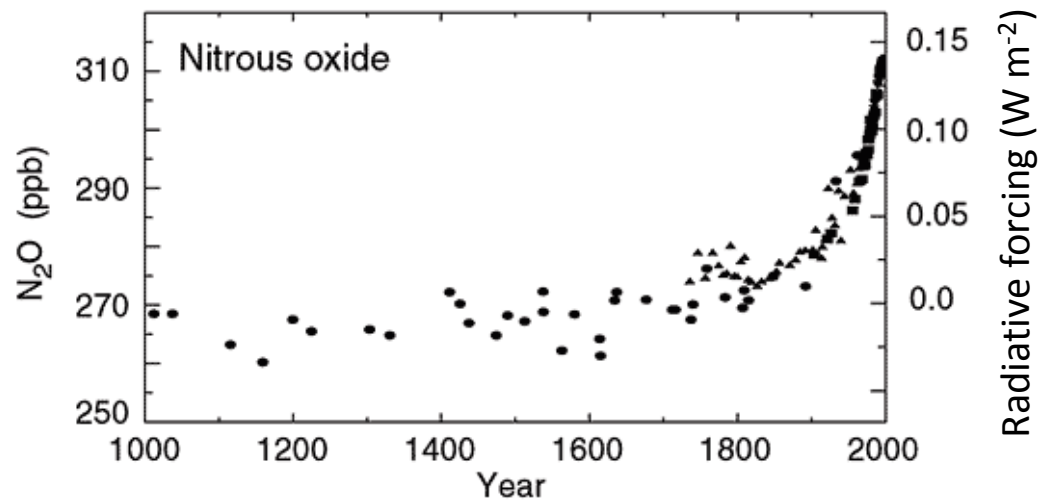
## Main Sink

Photochemical breakdown in stratosphere

## Atmospheric Lifetime

~130 years

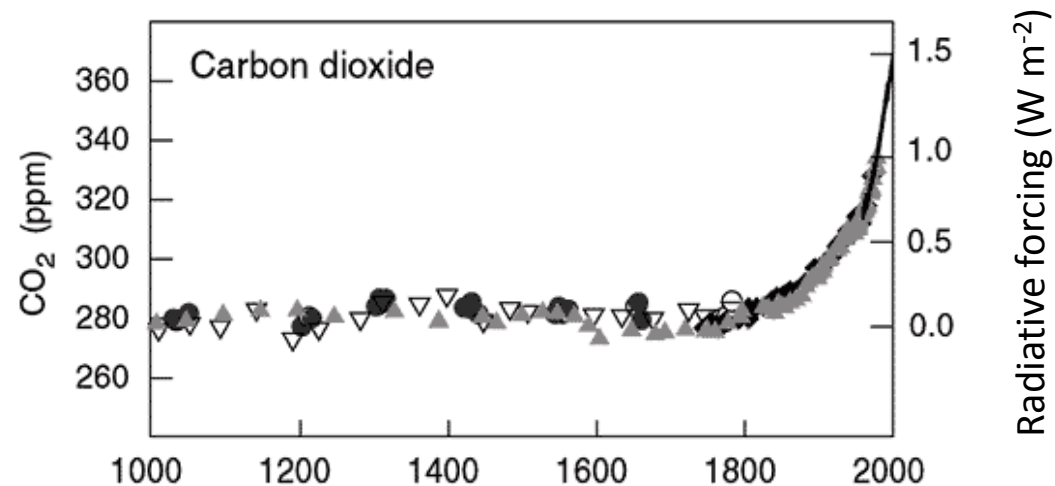
- Rise not well-approximated by quasi-steady relation.
- Sources have nearly doubled since preindustrial times.
- N<sub>2</sub>O likely to roughly double unless mitigation steps taken.



## Controls on CO<sub>2</sub> increase

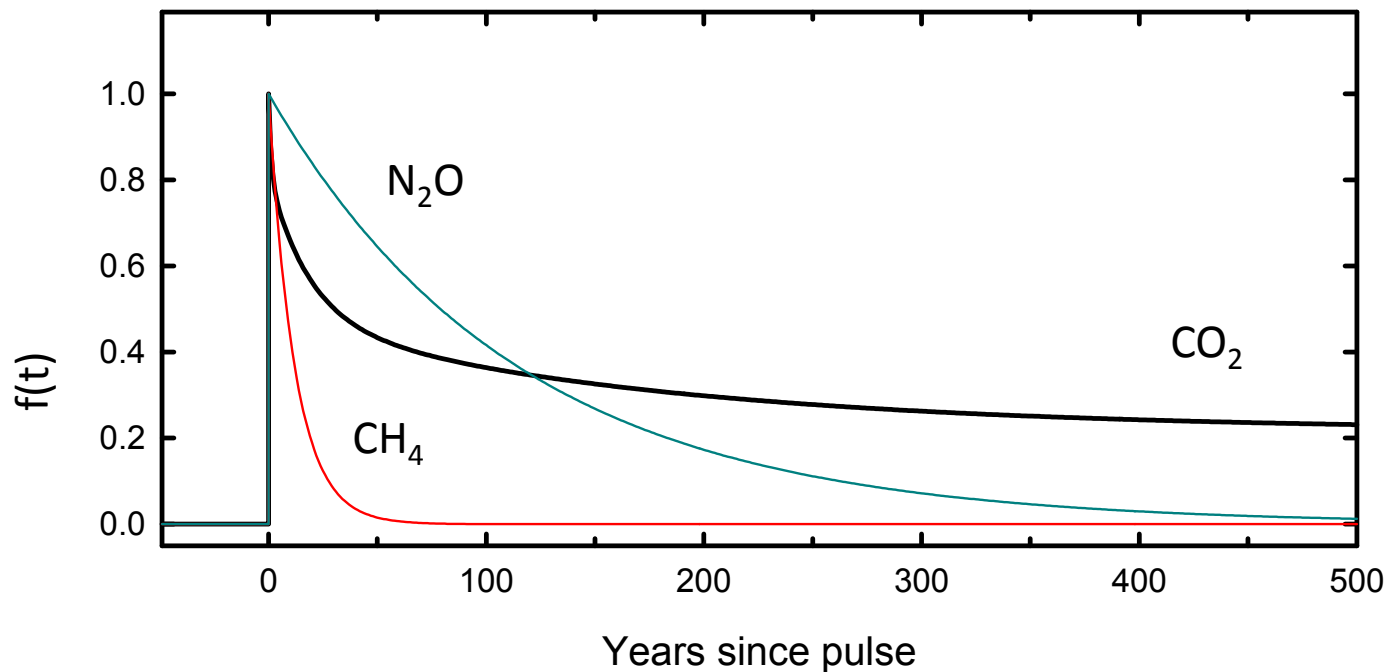
$$M_{air} \cdot \frac{dX_{CO_2}}{dt} = \text{Fossil-fuel emissions} + \text{Land use emissions} - \text{Ocean sink} - \text{Land sink}$$

- CO<sub>2</sub> doesn't have a well-defined atmospheric lifetime.
- CO<sub>2</sub> added to the atmosphere spreads to land and ocean carbon reservoirs.
- Carbon is conserved in this process.



# Pulse response function

If you emit a pulse of excess greenhouse gas, how does the excess evolve over time? The answer can be expressed as a function  $f(t)$  which starts off at unity, and decays with time, e.g.  $\text{CO}_2(t) = \text{CO}_2(0) * f(t)$



For  $\text{N}_2\text{O}$  and  $\text{CH}_4$ , the response function is a simple exponential:  
 $f(t) = e^{-kt}$ , where  $1/k$  is the atmospheric lifetime (8 yrs for  $\text{CH}_4$ , 130 yrs for  $\text{N}_2\text{O}$ )

For  $\text{CO}_2$ ,  $f(t)$  is a more complex function which drops to 0.5 after ~50 years, but drops more slowly after that.

# Global Warming Potential (GWP)

Regulation of greenhouse emissions requires a metric for assessing the relative impact of one gas versus another.

Relative impacts depend both on the radiation properties of the gas and on effective atmospheric lifetimes.

Key question: How to scale impacts in near-term versus distant future?  
Requires "discounting", in economic sense.

IPCC has adopted the GWP metric, which compares the impact of a ton of a particular greenhouse gas versus a ton of CO<sub>2</sub>. For example, for CH<sub>4</sub>:

$$GWP(\text{CH}_4) = \frac{\int_0^{\text{time horizon}} a_{\text{CH}_4} \times \text{CH}_4(t) dt}{\int_0^{\text{time horizon}} a_{\text{CO}_2} \times \text{CO}_2(t) dt}$$

$a_{\text{CH}_4}, a_{\text{CO}_2}$	Radiation efficiency of CO <sub>2</sub> or CH <sub>4</sub> (W m <sup>-2</sup> ppb <sup>-1</sup> )
CH <sub>4</sub> (t), CO <sub>2</sub> (t)	Time-dependent response (above baseline) to a 1 Mg pulse of CH <sub>4</sub> or CO <sub>2</sub> .
time horizon	Accounts for "discounting": no discounting until time horizon, after which all impacts are neglected.



# Global Warming Potential (GWP)

	Radiative forcing 1750-2005 (W m <sup>-2</sup> )	Life-time (years)	Radiation Efficiency (W m <sup>-2</sup> ppb <sup>-1</sup> )	GWP for Given Time Horizon		
				20-yr	100-yr	500-yr
Carbon Dioxide (CO <sub>2</sub> )	1.66	N/A	1.4x10 <sup>-5</sup>	1	1	1
Methane (CH <sub>4</sub> )	0.48	12	3.7x10 <sup>-4</sup>	72	25	7.6
Nitrous Oxide (N <sub>2</sub> O)	0.16	114	3.03x10 <sup>-3</sup>	289	289	153
CFC-12	0.17	100	0.32	11,000	10,900	5,200
CFC-11	0.063	45	0.25	6,730	4,750	1,620
SF <sub>6</sub>	0.0029	3,200	0.52	16,300	22,800	32,600
CF <sub>4</sub>	0.0034	50,000	0.1	5210	7390	11,200

Based on IPCC AR4, Tables 2.1 and 2.14

## Example Problem 1

What is time history of radiative forcing caused by emitting a one-time pulse of 200 Tg of CH<sub>4</sub>?

### Background information

Total moles dry air:  $1.77 \times 10^{20}$  moles

CH<sub>4</sub> radiative efficiency:  $3.7 \times 10^{-4} \text{ W m}^{-2} \text{ ppb}^{-1}$

CH<sub>4</sub> lifetime: 12 years

**First**, calculate the initial concentration excess in the air, assuming it's uniformly mixed:

Initial CH<sub>4</sub> excess as dry air mole fraction:

$$\frac{(200 \times 10^{12} \text{ g})}{(16 \text{ g mole})(1.77 \times 10^{20} \text{ moles})} = 7.1 \times 10^{-8} = 71 \text{ ppb}$$

## Example Problem 1, Continued..

**Second**, calculate CH<sub>4</sub> excess versus time. Here it is sufficient to note that the CH<sub>4</sub> excess will decay away exponentially according to the CH<sub>4</sub> lifetime of 12 years:

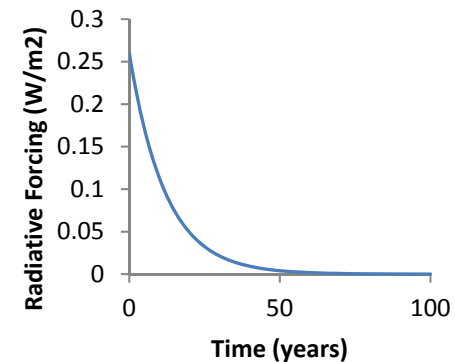
$$\text{CH}_4(t) = \text{CH}_4(0) \cdot e^{-t/12} \quad \text{where } \text{CH}_4(0) = 71 \text{ ppb}$$

Here  $t$  is the time since the pulse was emitted in years.

**Third**, calculate radiative forcing versus time. This is just the previous result multiplied by the radiative efficiency:

$$\text{RF}(t) = (3.7 \times 10^{-4})(71) e^{-t/12} = 0.26 e^{-t/12} \quad (\text{W m}^{-2})$$

Which holds for  $t > 0$ .



## Example Problem 2

**What is the average radiative forcing for our methane pulse over a 20 year time horizon?**

To calculate this, we simply need to integrate our previous expression from  $t = 0$  to  $t = 20$  years.

$$\begin{aligned} \text{Average forcing} &= \frac{0.26}{20} \int_0^{20} e^{-t/12} dt \\ &= \frac{0.26}{20} \cdot (-12) \cdot e^{-t/12} \Big|_0^{20} \\ &= -0.156 \cdot (e^{-20/12} - 1) \\ &= 0.156 \cdot (1 - e^{-20/12}) \\ &= 0.126 \text{ W m}^{-2} \end{aligned}$$

# Summary

- Relative contributions to radiative forcing, from GHGs, aerosols, etc.
- Non-CO<sub>2</sub> GHG buildup controlled by sources and "lifetime"
- GWP: combines radiation properties and atmospheric lifetimes.

## Appendix: Integration of non-CO<sub>2</sub> GHG equation for a pulse source on top of a steady source

$$M_{air} \cdot \frac{dX}{dt} = S(t) - k \cdot M_{air} \cdot X \quad (1)$$

$$S(t) = S_0 + S' \cdot \delta(t) \quad (2)$$

Here  $S_0$  is the steady source, and  $S'$  is the pulse.  $\delta(t)$  is the Dirac delta function. We further divide the solution into a steady state part  $X_0$ , and a perturbation  $X'$ :

$$X = X_0 + X' \quad (3)$$

$$S_0 = k \cdot M_{air} \cdot X_0 \quad (4)$$

Substituting (3) into (1) yields:

$$M_{air} \cdot \frac{d(X_0 + X')}{dt} = S_0 + S' \delta(t) - k \cdot M_{air} \cdot (X_0 + X') \quad (5)$$

Using (4) yields:

$$M_{air} \cdot \frac{dX'}{dt} = S' \delta(t) - k \cdot M_{air} \cdot X' \quad (6)$$

# Integration of GHG equation for pulse source added on top of a steady source

Combining (4) and (6) yields

$$M_{air} \cdot \frac{dX'}{dt} = S' \delta(t) - k \cdot M_{air} X' \quad (6)$$

For  $t > 0$  this is just

$$\frac{dX'}{dt} = -k \cdot X' \quad (7)$$

Which can be simply integrated to yield

$$X' = C e^{-kt} \quad (8)$$

Where C is an integration constant that we still need to determine.

# Integration of GHG equation for pulse source added on top of a steady source

Eq (8) holds for  $t > 0$ . For  $t < 0$  the solution is just  $X' = 0$ . The solution  $X'(t)$  is discontinuous at  $t = 0$  because of the pulse source, which boosts  $X'$  suddenly followed by exponential decay.  $C$  is the value of  $X'$  immediately after the pulse, i.e.  $C = X'(\delta t)$  at the limit  $\delta t$  approaches zero.

To determine  $C$ , we simply integrate (6) over a very small time window from  $t = 0$  to  $t = \delta t$

$$\int_0^{\delta t} M_{air} \cdot \frac{dX'}{dt} dt = \int_0^{\delta t} S' \delta(t) dt - \int_0^{\delta t} k \cdot M_{air} X' dt \quad (9)$$

The last term on the right vanishes at the limit  $\delta t \rightarrow 0$ , leaving

$$M_{air} \cdot (X'(\delta t) - X'(0)) = S' \quad (10)$$

With  $X'(0) = 0$ , we have

$$X'(\delta t) = S' / M_{air} = C \quad (11)$$

So the full solution for  $X'$  is

$$X' = (S' / M_{air}) e^{-kt} \quad (12)$$