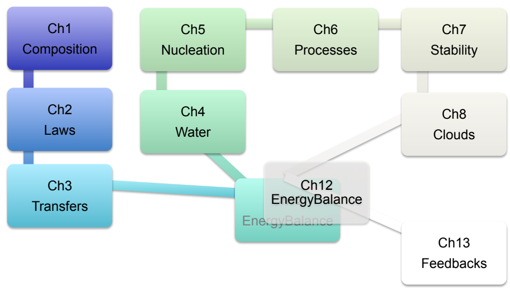
## Atmospheric Thermodynamics




## Climate Sciences: Atmospheric Thermodynamics

Instructor: Lynn Russell, NH343

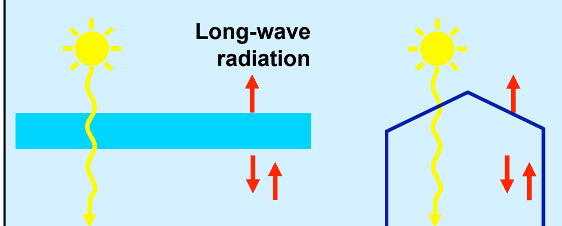
http://aerosol.ucsd.edu/courses.html

Text: Curry & Webster

## Course Principles

- Green classroom
  - Minimal handouts, optional paper text, etc.
- Respect for learning
  - On time, on schedule: quizzes
  - No chatting (in class), no cheating
- Focused exams
  - Core principles not algebra
- "Friday classes" – help on homework, projects
- Team learning by projects
  - Bring different backgrounds to common topics

## The Greenhouse Effect



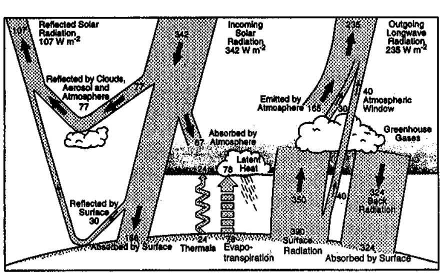
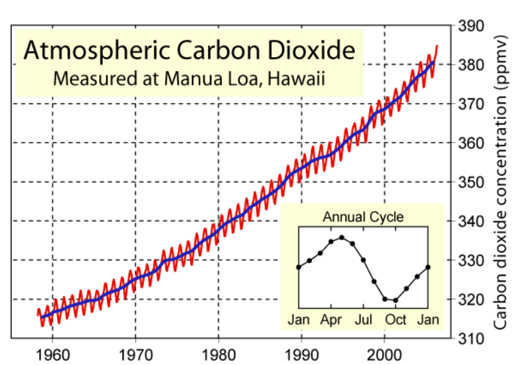




**Figure 12.2** Estimated annual mean global energy balance for the Earth. Units are W m$^{-2}$ (Kiehl and Trenberth, 1997).



## Review from Ch. 1

- Thermodynamic quantities
- Composition
- Pressure
- Density
- Temperature
- Kinetic Theory of Gases

Curry and Webster, pp. 1-17
Feynman, Book I, ch. 39

- Homework problem Ch. 2, Prob. 2 (due Monday 10/8; recitation on Friday 10/5)

## Thermal Structure of the Atmosphere

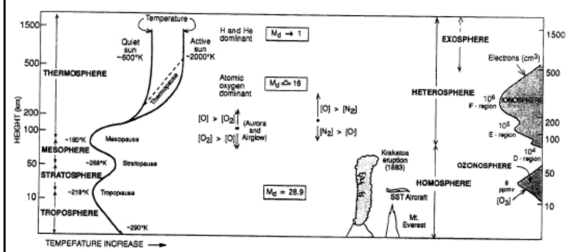
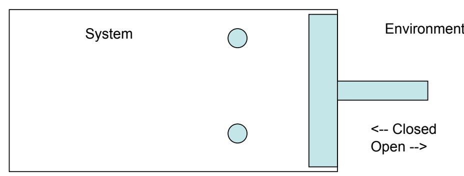



Figure 3.1. Two ways of dividing the atmosphere: by temperature structure (left side) and by composition (right side). The change in the apparent molecular weight of air ($M_d$) due to the changing composition of the atmosphere with height is shown in the center.

## Thermodynamic Quantities

- Classical vs. Statistical thermodynamics
- Open/closed systems
- Equation of state f(P,V,T)=0
- Extensive/intensive properties
- Thermal, engine, heat/work cycles




Intensive quantities: P, T, v, n
Extensive quantities: V, N

Concentration: n=N/V
Volume: v=V/N

## Composition

- Structure
  - Comparison to other planets
- $N_2$, $O_2$, Ar, $CO_2$, $H_2O$: 110 km constitute 99%
- Water, hydrometeors, aerosol

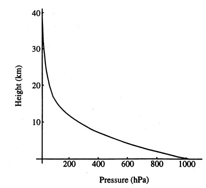
**Table 1.1** Main gaseous constituents of air, relative to the percent composition of dry air.

| Constituent | Formula | Molecular weight | % by volume | % by mass |
|---|---|---|---|---|
| Nitrogen | $N_2$ | 28.016 | 78.08 | 75.51 |
| Oxygen | $O_2$ | 31.999 | 20.95 | 23.14 |
| Argon | Ar | 39.948 | 0.93 | 1.28 |
| Carbon dioxide | $CO_2$ | 44.010 | 0.03 | 0.05 |
| Water vapor | $H_2O$ | 18.005 | 0–4 | |

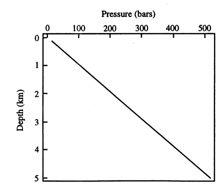
# Pressure

- Force per unit area
- 1 bar = $10^5$ Pa; 1 mb = 1 hPa; 1 atm = 1.013 bar
- Atmosphere vs. Ocean

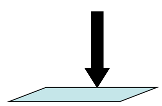
Atmosphere




Ocean



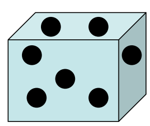

# Pressure



- Force per unit area
  - 1 bar = $10^5$ Pa;
  - 1 mb = 1 hPa;
  - 1 atm = 1.013 bar

# Density

- Specific volume: v=V/m
  - 0.78 $m^3$ $kg^{-1}$ for air
- Density: $\rho$=m/V
  - 1.29 kg $m^{-3}$ for air



# Temperature

- "Zeroeth" Law of Thermodynamics
  - Equilibrium of two bodies with third
  - Allows universal temperature scale
- Temperature scale
  - Two fixed points: Kelvin, Celsius
  - Thermometer
- Lapse Rate $\Gamma = -\partial T/\partial z$
  - Change in temperature with altitude
  - Typically $\Gamma$=6.5 K/km
- Temperature inversion $\Gamma$<0
  - Boundary layer "cap"
  - Tropopause between troposphere and stratosphere

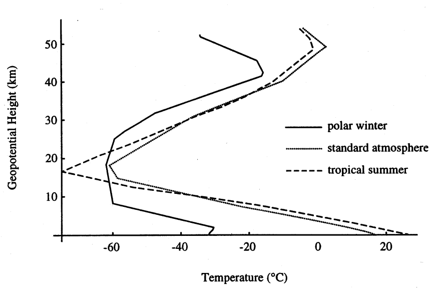



**Figure 1.7** Vertical temperature structure in the atmosphere below about 50 km. Temperature decreases with height in the troposphere, except for the polar winter, where surface temperatures are very low, causing a temperature inversion near the surface (U.S. Standard Atmosphere, 1976).

# History of the Standard Atmosphere

- With a little digging, you can discover that the Standard Atmosphere can be traced back to 1920. The constant lapse rate of 6.5° per km in the troposphere was suggested by Prof. Toussaint, on the grounds that
  - … what is needed is … merely a law that can be conveniently applied and which is sufficiently in concordance with the means adhered to. By this method, corrections due to temperature will be as small as possible in calculations of airplane performance, and will be easy to calculate. …
  - The deviation is of some slight importance only at altitudes below 1,000 meters, which altitudes are of little interest in aerial navigation. The simplicity of the formula largely compensates this inconvenience.
- The above quotation is from the paper by Gregg (1920). The early motivations for this simplified model were evidently the calibration of aneroid altimeters for aircraft, and the construction of firing tables for long-range artillery, where air resistance is important.
- Unfortunately, it is precisely the inaccurate region below 1000 m that is most important for refraction near the horizon. However, the Toussaint lapse rate, which Gregg calls "arbitrary", is now embodied in so many altimeters that it cannot be altered: all revisions of the Standard Atmosphere have preserved it.
- Therefore, the Standard Atmosphere is really inappropriate for astronomical refraction calculations. A more realistic model would include the diurnal changes in the boundary layer; but these are still so poorly understood that no satisfactory basis seems to exist for realistic refraction tables near the horizon.

http://mintaka.sdsu.edu/GF/explain/thermal/std_atm.html

# International Standard Atmosphere

- The ISA model divides the atmosphere into layers with linear temperature distributions.[2] The other values are computed from basic physical constants and relationships. Thus the standard consists of a table of values at various altitudes, plus some formulas by which those values were derived. For example, at sea level the standard gives a pressure of 1.013 bar and a temperature of 15°C, and an initial lapse rate of −6.5 °C/km. Above 12km the tabulated temperature is essentially constant. The tabulation continues to 18km where the pressure has fallen to 0.075 bar and the temperature to −56.5 °C.[3][4]

**Layers in the ISA**

| Layer | Level Name | Base Geopotential Height $h$ (in km) | Base Geometric Height $z$ (in km) | Lapse Rate (in °C/km) | Base Temperature $T$ (in °C) | Base Atmospheric Pressure $p$ (in Pa) |
|---|---|---|---|---|---|---|
| 0 | Troposphere | 0.0 | 0.0 | -6.5 | +15.0 | 101,325 |
| 1 | Tropopause | 11.000 | 11.019 | +0.0 | -56.5 | 22,632 |
| 2 | Stratosphere | 20.000 | 20.063 | +1.0 | -56.5 | 5,474.9 |
| 3 | Stratosphere | 32.000 | 32.162 | +2.8 | -44.5 | 868.02 |
| 4 | Stratopause | 47.000 | 47.350 | +0.0 | -2.5 | 110.91 |
| 5 | Mesosphere | 51.000 | 51.413 | -2.8 | -2.5 | 66.939 |
| 6 | Mesosphere | 71.000 | 71.802 | -2.0 | -58.5 | 3.9564 |
| 7 | Mesopause | 84.852 | 86.000 | — | -86.2 | 0.3734 |


* U.S. Extension to the ICAO Standard Atmosphere, U.S. Government Printing Office, Washington, D.C., 1958.
* U.S. Standard Atmosphere, 1962, U.S. Government Printing Office, Washington, D.C., 1962.
* U.S. Standard Atmosphere Supplements, 1966, U.S. Government Printing Office, Washington, D.C., 1966.
* U.S. Standard Atmosphere, 1976, U.S. Government Printing Office, Washington, D.C., 1976.


# Geopotential Height

**Geopotential height** is a vertical coordinate referenced to Earth's mean sea level — an adjustment to geometric height (elevation above mean sea level) using the variation of gravity with latitude and elevation. Thus it can be considered a "gravity-adjusted height." One usually speaks of the geopotential height of a certain **pressure level**, which would correspond to the geopotential height necessary to reach the given pressure.

At an elevation of $h$, the **geopotential** is defined as

$$\Phi = \int_0^h g(\phi, z)\, dz\,,$$

where $g(\phi,z)$ is the acceleration due to gravity, $\phi$ is latitude, and $z$ is the geometric elevation.

Thus, it is the gravitational potential energy per unit mass at that level. The **geopotential height** is

$$Z_g = \frac{\Phi}{g_0}\,,$$

where $g_0$ is the standard gravity at mean sea level.

Geophysical scientists often use geopotential height rather than geometric height, because doing so in many cases makes analytical calculations more convenient. For example, the primitive equations which weather forecast models solve are more easily expressed in terms of geopotential than geometric height. Using the former eliminates centrifugal force and air density (which is very difficult to measure) in the equations.

# ICAO Standard Atmosphere

The International Civil Aviation Organizintion (ICAO) Standard Atmosphere gives the average values for meteorological element at 40°N from mean sea level (MSL) to 80km (262,500 ft).

The ICAO Standard Atmosphere does not contain water vapour

Some of the values defined by ICAO are:

**ICAO Standard Atmosphere**

| Height km & ft | Temperature °C | Pressure hPa | Lapse Rate °C/1000ft |
|---|---|---|---|
| 0km MSL | 15.0 | 1013.25 | 1.98 (Tropospheric) |
| 11km 36,000ft | -56.5 | 226.00 | 0.00 (Stratospheric) |
| 20km 65,000ft | -56.5 | 54.70 | -1.00 (Stratospheric) |
| 32km 105,000ft | -44.5 | 8.68 | |

As this is a Standard, you will never encounter these conditions outside of a laboratory, but many Aviation standards and flying rules are based on this, altimetry being a major one. The standard is very useful in Meteorology for comparing actual values to.

# Hydrostatic Balance

- Applicable to most atmospheric situations (except fast accelerations in thunderstorms)

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\partial p = -\frac{pg}{R_d T}\partial z$$

Curry and Webster, Ch. 1

# Homogeneous Atmosphere

- Density is constant
- Surface pressure is finite
- Scale height H gives where pressure=0

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$dp = -\rho g dz$$

$$\int_{p_0}^{0} dp = -\int_0^H \rho g dz$$

$$0 - p_0 = -(\rho g H - 0)$$

$$p_0 = \rho g H$$

$$H = \frac{p}{\rho g} = \frac{R_d T_0}{g}$$

Curry and Webster, Ch. 1

# Hydrostatic + Ideal Gas + Homogeneous

- Evaluate lapse rate by differentiating ideal gas law

Ideal gas

$$p = \rho R_d T$$

Density constant

$$\frac{\partial p}{\partial z} = \rho R_d \frac{\partial T}{\partial z}$$

$$\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}\right) = R_d\left(-\frac{\partial T}{\partial z}\right)$$

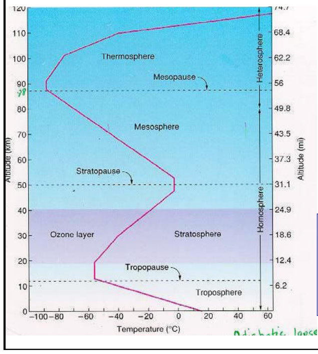
Hydrostatic

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\Gamma = -\frac{\partial T}{\partial z} = \frac{g}{R_d} = 34.1^{\circ}\mathrm{C/km}$$

Curry and Webster, Ch. 1

# Lapse Rate




- $\Gamma$ = -dT/dz
- Change in temperature with height
- Defined to be positive in troposphere

Troposphere:
dT = 70 C
dz = 10 km

So, $\Gamma$ = 7 C/km

# Hydrostatic Equation (1)

- Hydrostatic Balance (1.33) $g = -\frac{1}{\rho}\frac{\partial p}{\partial z}$
- Geopotential Height (1.36a) $Z = -\frac{1}{g_0}\int_0^z g dz$
- Homogeneous atmosphere (1.38)

$$p_0 = \rho g H$$
$$[\text{N.B. } \rho = \text{constant}]$$
$$H = \frac{R_d T_0}{g} = 8 \text{ km}$$

$$\frac{\partial p}{\partial z} = \rho R_d \frac{\partial T}{\partial z}$$
$$[\text{N.B. ideal gas}]$$
$$\Gamma = -\frac{\partial T}{\partial z} = \frac{g}{R_d} = 34.1^{\circ}\text{C km}^{-1}$$

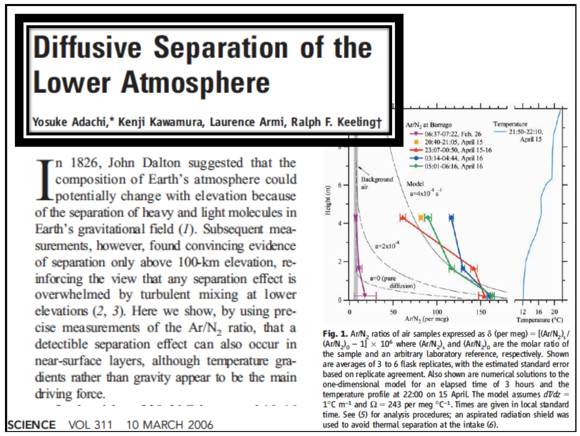
# Hydrostatic Equation (2)

- Isothermal Atmosphere (1.42)

$$\partial p = -\frac{pg}{R_d T}\partial z$$
$$[\text{N.B. T} = \text{constant}]$$
$$p = p_0 \exp\left(-z/_H\right) \text{ for } H = RT/_g$$

- Constant Lapse Rate (1.48)

$$\frac{dp}{p} = -\frac{g}{R_d}\frac{dz}{T_0 - \Gamma z}$$
$$[\text{N.B. } \Gamma = \text{constant}]$$
$$p = p_0\left(\frac{T}{T_0}\right)^{g/_{R_d \Gamma}}$$



**Diffusive Separation of the Lower Atmosphere**

Yosuke Adachi,* Kenji Kawamura, Laurence Armi, Ralph F. Keeling†

In 1826, John Dalton suggested that the composition of Earth's atmosphere could potentially change with elevation because of the separation of heavy and light molecules in Earth's gravitational field (*1*). Subsequent measurements, however, found convincing evidence of separation only above 100-km elevation, reinforcing the view that any separation effect is overwhelmed by turbulent mixing at lower elevations (*2*, *3*). Here we show, by using precise measurements of the $Ar/N_2$ ratio, that a detectible separation effect can also occur in near-surface layers, although temperature gradients rather than gravity appear to be the main driving force.



**Fig. 1.** $Ar/N_2$ ratios of air samples expressed as $\delta$ (per meg) = $[(Ar/N_2)_s/(Ar/N_2)_0 - 1] \times 10^6$ where $(Ar/N_2)_s$ and $(Ar/N_2)_0$ are the molar ratio of the sample and an arbitrary laboratory reference, respectively. Shown are averages of 3 to 6 flask replicates, with the estimated standard error based on replicate agreement. Also shown are numerical solutions to the one-dimensional model for an elapsed time of 3 hours and the temperature profile at 22:00 on 15 April. The model assumes $dT/dz$ = 1°C m$^{-1}$ and $\Omega$ = 243 per meg °C$^{-1}$. Times are given in local standard time. See (*5*) for analysis procedures; an aspirated radiation shield was used to avoid thermal separation at the intake (*6*).

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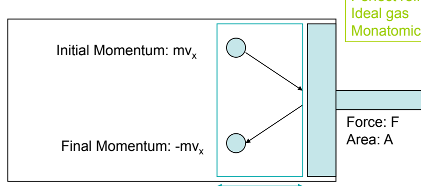
# Kinetic Theory of Gases

- What is the pressure of a gas?
- What is the temperature of a gas?
- Pressure-volume-temperature relationship(s)

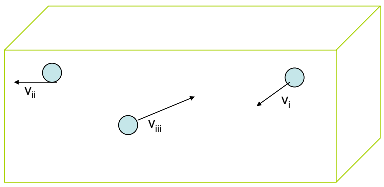
- How does pressure (and volume) relate to energy?
- Kinetic energy
- Internal energy
  - The "fine print"

Individual collisions
Perfect reflection
Ideal gas
Monatomic gas



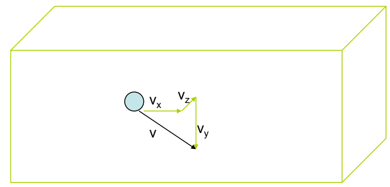

If all atoms had same x-velocity $v_x$:
Momentum Change for one Atom-Collision: [Initial]-[Final] = $mv_x$-($-mv_x$) = $2mv_x$
Number of Atom-Collisions-Per-Time: [Concentration]*[Volume] = [n]*[$v_x$A]
Force = [Number]*[Momentum Change] = [$nv_x$A]*[$2mv_x$] = $2nmAv_x^2$
Pressure = [Force]/[Area] = $2nmv_x^2$

For atoms with average velocity-squared of $\langle v_x^2 \rangle$:
Pressure = [Force]/[Area] = $nm\langle v_x^2 \rangle$



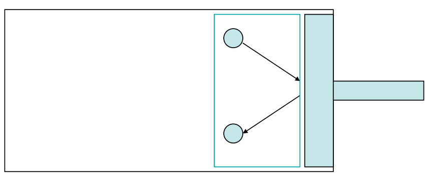

Population-averaged Velocity: $<v^2>=[v_i^2 + v_{ii}^2 + v_{iii}^2 + \ldots + v_n^2]/n$
Scalar multipliers: $<mv^2/2>=[mv_i^2 + mv_{ii}^2 + mv_{iii}^2 + \ldots + mv_n^2]/2n$

How many will hit "right" wall? n/2




3D velocity: $<v^2>=<v_x^2>+<v_y^2>+<v_z^2>$
Random motion (no preferred direction): $<v_x^2>=<v_y^2>=<v_z^2>$

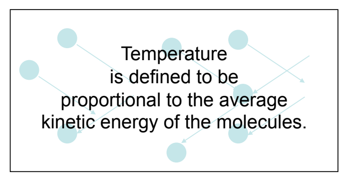
$<v_x^2>= <v^2>/3$



P $= nm<v_x^2>$
$=[2/2]*[nm]*[<v^2>/3]$
$=[2/3]n*<mv^2/2>$
=[2/3]n*[kinetic energy of molecule]

PV $=[2/3]*[N*<mv^2/2>]$
$=[2/3]*U$
$=[2/3]*E_k$

Concentration: n=N/V
Total "internal" energy: U
Kinetic energy of gas




PV $=[2/3]*E_k$
$E_k$ $=[3/2]*$ PV

Define $T = f(E_k)$
For scale choose $T=(2/3Nk)*E_k$
$E_k$ $=(3/2)*NkT$

Then PV = NkT = nR*T

Kinetic energy of gas
RHS is independent of gas
--> so scale can be universal

Mean k.e.: $E_k/N=(3/2)kT$
$k=1.38\times10^{-23}$ J/K

$R^*=N_0k=8.314$ J/mole/K