

e.

SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2015 Midterm Exam (No calculators, notes, books, PDAs.) **KEY**
Curry and Webster, Ch. 1-4 (and Section 12.1)

Here are some numerical values, some of which may be useful on this exam:

Average radius of Earth: 6370 km

Mean reflectivity of the Earth: 0.31

Mean molecular weight of dry air: 29 g/mole

Mean molecular weight of water vapor: 18 g/mole

Gas constant for dry air, R_d : 287 J deg⁻¹ kg⁻¹

Gas constant for water vapor, R_v : 461 J deg⁻¹ kg⁻¹

Specific heat at constant pressure, c_p : 1004 J deg⁻¹ kg⁻¹

Specific heat at constant volume, c_v : 717 J deg⁻¹ kg⁻¹

Latent heat of vaporization for water at 273K, L_v : 2.5×10^6 J kg⁻¹

Solar luminosity: 3.92×10^{26} W

Earth-sun distance: 1.50×10^{11} m

Stefan-Boltzmann constant, σ : 5.67×10^{-8} W m⁻² K⁻⁴

5. 1. The Earth's atmosphere provides a greenhouse effect that makes the planet warmer than it would be with no atmosphere.

a. Describe how a greenhouse (with a glass roof) works to cause warming, noting in particular what happens to (i) incoming and outgoing energy and (ii) whether each is shortwave or longwave. Specify whether energy is transmitted, reflected, absorbed or emitted in each direction.

Greenhouses work because the Glass on a greenhouse (i) transmits incoming shortwave radiation and (ii) absorbs then emits outgoing longwave radiation.

b. What components of the atmosphere cause the greenhouse effect? Name at least three.

Water vapor, CO₂, CH₄, N₂O, halocarbons.

c. Calculate the amount of incoming solar radiation at the top of the atmosphere [in W m⁻²].

Instantaneous at solar noon = (luminosity)/(4π*ESdistance²)

$$= (3.92 \times 10^{26}) / (4 * 3.14 * (1.50 \times 10^{11})^2) = 1390 \text{ W m}^{-2};$$

$$\text{Averaged over Earth surface} = (1390 \text{ W m}^{-2}) / 4 = 342 \text{ W m}^{-2}.$$

d. Use a simple model **with no atmosphere** to calculate what the Earth's temperature would be if there were no greenhouse effect at all (but the albedo was still 0.3). Give the equations in your model and state its assumptions.

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$$F_L = \sigma T_{\text{atm}}^4 \text{ (assumption 1; Eqn. 3.20)}$$

$$F_S = F_L \text{ (assumption 2-3; Eqn. 3.20)}$$

$$0.25 * S_0 (1 - \alpha_p) = \sigma T_{\text{atm}}^4 \text{ (Eqn. 3.20, Eqn. 12.)}$$

$$T_{\text{atm}} = 255 \text{ K}$$

where $S_0 = L_0 / (4\pi d^2) = 1.3938 \times 10^3 \text{ W m}^{-2}$ (Eqn. 12.), $\alpha_p = 0.3$, $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

1. Consider the properties of the standard atmosphere, assuming hydrostatic balance and constant lapse rate of $\Gamma=6.5\text{K km}^{-1}$.
 - a. Write the equation for the hydrostatic balance and identify which two forces are “balanced” to give this equation.
 - b. Derive an expression for the variation of pressure with altitude, $p(z)$, in terms of the altitude z , surface pressure p_0 , surface temperature T_0 and a constant lapse rate Γ .
 - c. What are typical values for p_0 and T_0 for modern Earth?
 - a. $dp=-\rho g dz$: The hydrostatic balance is the equality of upward (pressure gradient) and downward (gravitational) forces in the atmosphere that results in little net vertical motion.
 - b. From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{p g}{R_d T} \partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{p g}{R_d T} dz$$

$$\frac{dp}{p} = -\left(\frac{g}{(T_0 - \Gamma z) R_d}\right) dz$$

$$\int_{p_0}^p \frac{dp}{p} = \int_0^z \left(-\frac{g}{R_d}\right) \frac{dz}{(T_0 - \Gamma z)}$$

$$\ln \frac{p}{p_0} = \left(\frac{g}{\Gamma R_d}\right) \ln \frac{(T_0 - \Gamma z)}{T_0}$$

$$\left(\frac{p}{p_0}\right) = \left(\frac{T_0 - \Gamma z}{T_0}\right)^{\left(\frac{g}{R_d \Gamma}\right)} = \left(1 - \frac{\Gamma z}{T_0}\right)^{\left(\frac{g}{R_d \Gamma}\right)}$$

$$p = p_0 \left(1 - \frac{\Gamma z}{T_0}\right)^{\left(\frac{g}{R_d \Gamma}\right)}$$

c. $p_0 = 1013 \text{ mb}$; $T_0 = 288\text{K}$.

3. Give an equation for optical depth in terms of atmospheric properties. For a pathlength of 5 km, evaluate the optical depth for an average density of 1 kg m^{-3} and an average absorption coefficient of $2 \times 10^{-5} \text{ m}^2 \text{ kg}^{-1}$, what is the optical depth (assuming zenith angle of 0)?

$$d\tau_\lambda = k_\lambda^{abs} \rho dx \quad (3.25)$$

Eqn. 3.25: $d\tau = k^{abs} \rho dx$; so for average $k^{abs} \rho$, $\tau = k^{abs} \rho \Delta x$.
Optical depth $\tau = (2 \times 10^{-5} \text{ m}^2/\text{kg})(1 \text{ kg}/\text{m}^3)(5000 \text{ m}) = 0.1$

2. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
- mechanical equilibrium** occurs when two substances have no difference in pressure between one phase and another (or from one species to another), i.e. $p_1=p_2$.
 - ideal gas** is a vapor in which molecules collide with perfectly elastic collisions and without interactions, typically consistent with behavior at low pressures (<2 atm) and high temperatures ($>250\text{K}$); the equation of state is $pv=RT$.
 - Reversible work is work done in a process that proceeds with infinitesimally small steps that can be reversed at any step; it is equivalent to expansion work $dw=-pdv$.
 - Stefan-Boltzmann law** gives the irradiance of a black body as a function of temperature, i.e. $F=\sigma T^4$.
 - enthalpy** is the internal energy of the system, plus the capacity for pressure-volume work, i.e. $h=u+pv$.
 - entropy** is a state function which gives the maximum (reversible) work done for a state change, given by $d\eta=(dq/T)_{\text{rev}}$ (Eqn. 2.25a)
 - Gibbs' phase rule** says that for a system in thermal, chemical, and mechanical equilibrium, then the number of degrees of freedom are given by the Gibbs phase rule: $f=\chi-\varphi+2$ (for χ components in φ phases).
5. The saturation vapor pressure (of water) at a temperature of 30°C is 42.4 hPa. Consider moist air at 30°C , a pressure of 1000 hPa, and a relative humidity of **50%**. Find the values:
- vapor pressure
 $e=H*e_s = 0.5*42.4 = 21.2$ hPa [Eqn. 4.34a].
 - mixing ratio
 $w_v=m_v/m_d=(M_v/M_d)*(e/(p-e))=0.622*(21.2/(1000-21.2))$ [Eqn. 4.36].
 - specific humidity
 $q_v=m_v/(m_v+m_d)=w_v/(w_v+1)$, w_v from (b) [Eqn. 1.20].
 - virtual temperature
 $T_v=(1+0.608q_v)*T=(1+0.608q_v)*303$, q_v from (c) [Eqn. 1.25].
 - potential temperature**
 $\Theta=303*(p/p_0)^{(Rd/cpd)}=303*(1000/1000)^{(287/717)}=303\text{K}$ [Eqn. 2.62]
6. The saturation vapor pressure (of water) doubles approximately every 10°C in typical atmospheric conditions. This *strong dependence* of saturation vapor pressure on temperature (i.e. this large change of doubled pressure per 10°C increase in temperature) provides many of the unique cloud feedbacks that govern climate on Earth.
- Give **the first law of thermodynamics** (i) *for a change of internal energy* in terms of heat and work, i.e. $du=fcn(q,w)$ and, (ii) for a change of Gibbs free energy in terms of T, p, v , and η (entropy), i.e. $dg=fcn(T, p, v, \eta)$.
- (i) $du=dq+dw$; (ii) $dg=vd p - \eta dT$.
- Give the equation that describes the increase in saturation vapor pressure with temperature in (i) the *differential* form and (ii) the **integral** form.

This equation is the Clausius-Clapeyron equation;

(i) the differential form is given as:

$$\left(\frac{\partial p}{\partial T}\right)_g = \frac{L_v p}{R_v T^2}$$

(ii) the integral form is given as:

$$e_2 = e_1 \exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right] \quad \text{or} \quad p_2 = p_1 \exp\left[-\frac{L_{lv}}{R_v}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right]$$

(c) Identify (i) the water property *that appears in equations in (b)* that makes this dependence strong (Hint: The answer is a quantity related to water that is *not a pressure*.) and (ii) the property of the water molecule that results in (i) being large.

(i) Latent heat, L_{lv} , which is quite high for water, because (ii) water is a very polar molecule that forms strong intermolecular bonds.

6. The phase diagram of pure water is shown below. For these questions, assume that it describes (approximately) the three phases of water in the atmosphere.
- On the diagram, label the triple point of water. [See A on diagram below.](#)
 - How many degrees of freedom are there when two phases of pure water coexist? **1.**
 - Label the region(s) where liquid water exists with two degrees of freedom. [See C on diagram below.](#)
 - Identify the region on the diagram where liquid and vapor coexist. [See D on diagram below.](#)

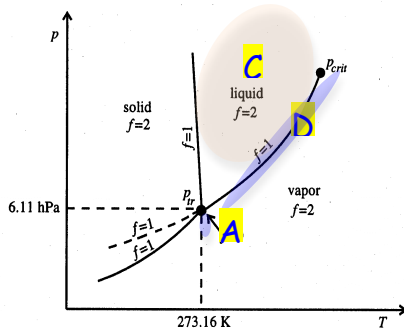


Figure 4.3 p, T phase diagram for water. The three curves indicate those points for which two phases coexist at equilibrium. The dashed curve is the extension of the vapor-pressure curve for liquid water to temperatures below 273.16 K. The solid curve below 273.16 K connects the points at which ice and vapor coexist at equilibrium. p_{crit} indicates the pressure and temperature values beyond which liquid water and water vapor are no longer distinguishable from one another. p_t indicates the triple point, the unique p, T point at which all three phases coexist.