Precipitation Growth Rate In Western Atlantic Cumulus Type Clouds

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1. Introduction

Cloud formation is a key aspect of the climate system, affecting the earth-atmosphere hydrological cycle, weather, and the global energy balance. Clouds are highly dynamic systems that are strongly controlled by thermodynamics. They consist of microscopic liquid droplets and ice crystals. According to Kohler theory, cloud droplets initially form by condensation of water vapor on to nuclei when relative humidity is higher than 100%. In warm clouds, larger droplets fall with higher velocity than smaller droplets because large droplets deform to become non spherical and more aerodynamic, but this study examines spherical droplets and particles. As a result the large droplets can fall, collide and combine with smaller droplets, and continue to grow. When drops become large enough so that their acceleration due to gravity is larger the acceleration due to drag force, they can fall to the earth as precipitation. Another important process that forms precipitation is aggregation. Aggregation happens when two solid snowflake collide and combine (Pruppacher 1997).

The growth time of droplets and snowflakes in the cloud is controlled by many thermodynamic parameters such as a cloud's temperature and pressure, supersaturation with respect to liquid and ice, and size distribution of droplets and snowflakes in the cloud. In this project, a simple model is used to calculate the growth time of droplets and snow and evaluate sensitivity of the growth time with respect to different microphysical properties of the cloud.

We specifically used the model to calculate the growth time of droplets and snow in western Atlantic cumulus and cumulonimbus clouds. Cumulus clouds are low-level clouds typically found at 2000-3000 ft that can have noticeable vertical development and clearly defined edges. They contain liquid water and generally do not produce precipitation except for brief showers from congestus cumulus. Congestus cumulus clouds grow with strong updraughts of warm moist air and form cumulonimbus clouds. Typical altitude of cumulonimbus clouds is 2000-45000 ft. Cumulonimbus contains liquid water throughout the cloud and ice crystals at the top (Obasi 1987).

2. Description of the model

The models for this project are based on the theory provided in Thermodynamics of Atmospheres and Oceans (Curry and Webster 1999). Three different models are considered for three cases of atmospheric precipitation growth. The three cases considered for modeling are liquid droplets, ice spheres, and aggregating snow. Each of the three models uses a slightly different form of theoretical equations (Curry and Webster 1999). For all of the models, the key assumption is that the temperature at which the droplet or ice particle grows remains constant. Therefore the temperature dependent atmospheric properties are also assumed to be constant. Additionally, the input variable, growth, is dependent on supersaturation values of the cloud. For a given case, the supersaturation value is also assumed to be constant throughout droplet/particle growth. These assumptions will produce some error as properties of clouds rarely remain constant and the error has not been quantified in this paper. Another limitation of the modeling is the assumption that the equations are valid for droplet/particle sizes from zero up to extremely large sizes. It is known (Curry and Webster 1999) that the growth behavior for very small droplets and particles do not follow these equations. Additionally, exceedingly large precipitation falls out of the clouds and does not continue to grow. The MATLAB code for each of the models is located in the Appendix.

The liquid droplet model uses Equation 1, which is provided by Curry and Webster (1999) and is given below. This equation dictates the relationship between droplet radius and growth time as a function of the atmospheric properties of supersaturation (S), latent heat (L_{lv}) of water, density of water (ρ_l), gas constant (R_v), temperature (T), saturation pressure (e_s), diffusion coefficient (D_v), and coefficient of atmospheric conductivity (κ). The definitions and physical meanings of these properties are provided in the next section. In addition to the assumptions listed above, curvature and solute effects have been ignored for this derivation as their effects are small once the drop size increases beyond a few microns (Curry and Webster 1999).

$$(Equation\ 1)\ \ r\frac{dr}{dt} = \frac{S-1}{\rho_l\left(\frac{L_{lv}^2}{\kappa R_v T^2} + \frac{R_v T}{e_s D_v}\right)}$$

Known properties from cumulus clouds were used to calculate the growth time of a liquid droplet. The results, in Figures 1-2 below, give radii as a function of growth time with consideration for small changes in temperature and supersaturation.

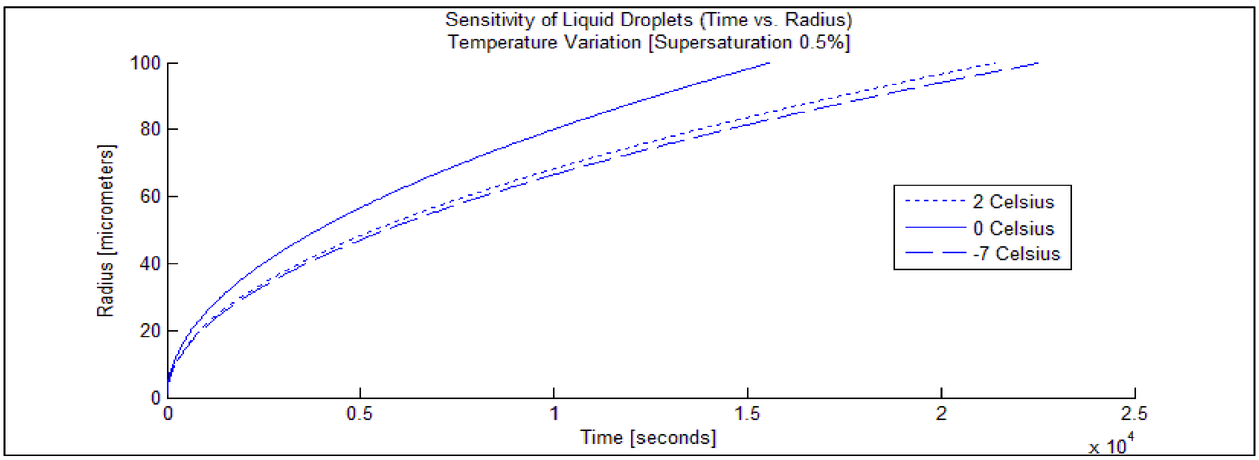

Figure 1. Liquid droplet growth as a function of time for three different temperatures common to cumulus clouds (parameters Abel et al. 2014).

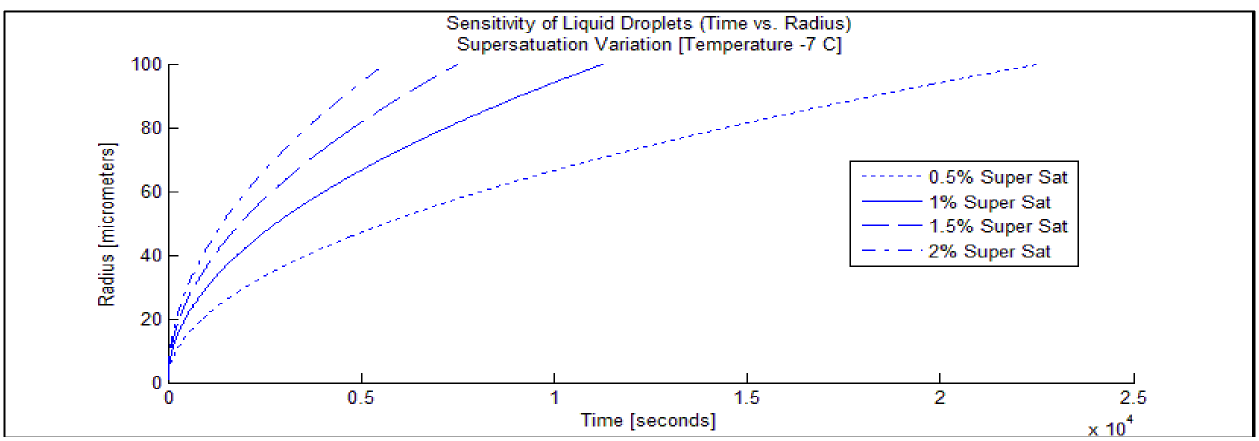

Figure 2. Liquid droplet growth as a function of time for different supersaturation values common to cumulus clouds (parameters Pruppacher 1997).

The model for spherical ice is very similar to the droplet case and is based on Equation 2 (Curry and Webster 1999). Additionally, the same assumptions of constant temperature and constant properties were used for this model.

$$(Equation\ 2)\ \frac{dm}{dt} = \frac{4\pi C(S_i - 1)}{\left(\frac{L_{iv}^2}{\kappa R_v T^2} + \frac{R_v T}{e_{si} D_v}\right)}$$

Similar to the liquid droplet calculations, Figures 3-4 shows the calculated radii versus time plot for the ice spheres. In addition to the general assumptions listed above, the ice crystal is assumed to develop in a spherical shape. This model also assumes the equation holds valid from a very small diameter. However, the growth rate at very small diameters may be only half the rate predicted by this model (Curry and Webster 1999). - I got up to here.

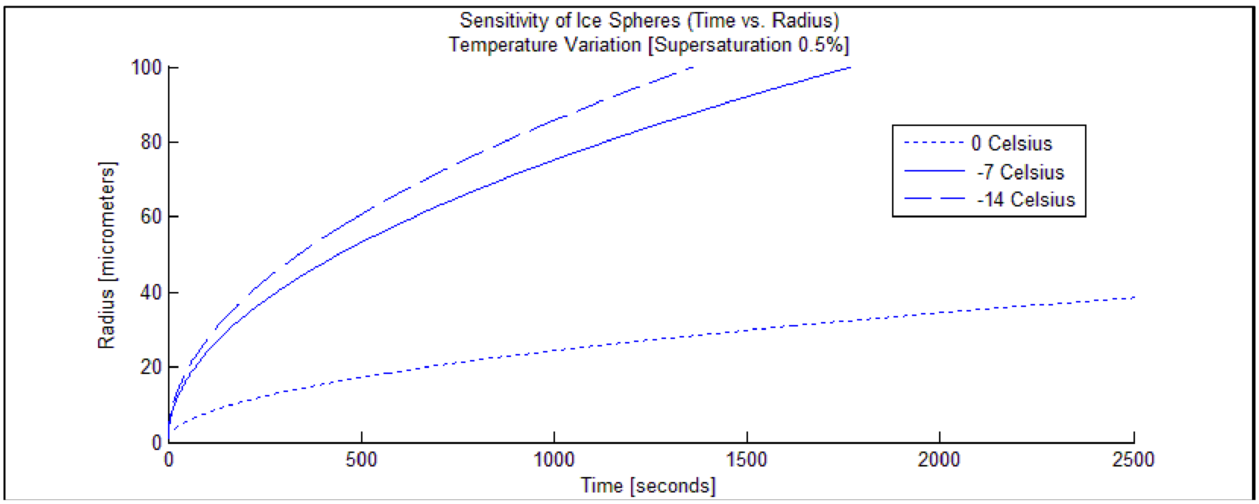

Figure 3. Ice sphere growth as a function of time for three different temperatures common to cumulus clouds (parameters from Abel et al. 2014).

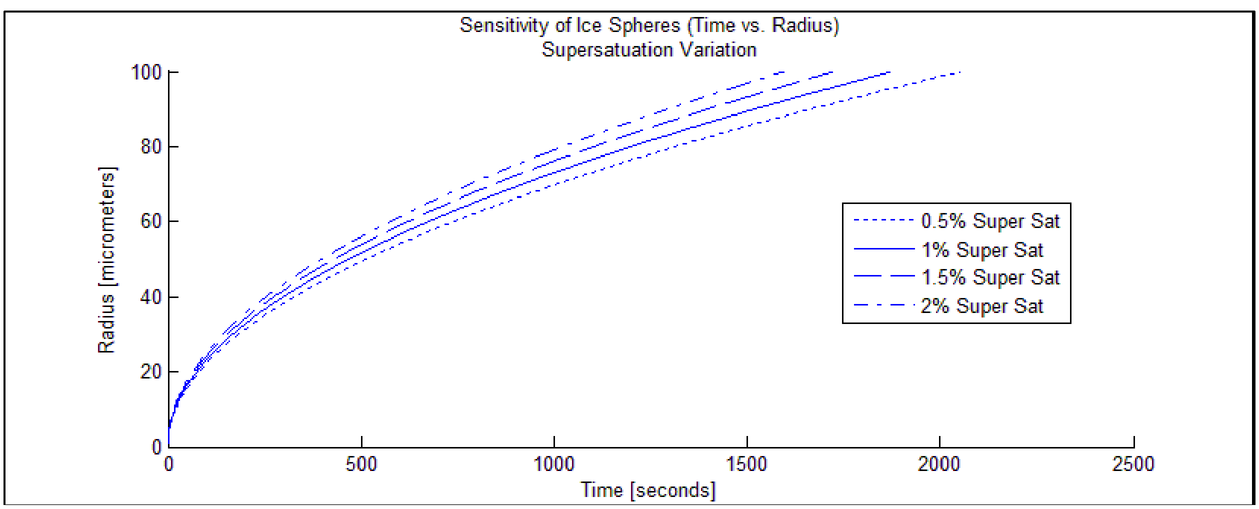

Figure 4. Ice sphere growth as a function of time for different supersaturation values common to cumulus clouds (parameters from Pruppacher 1997).

The third model is for aggregate snow growth and is based on Equations 3 and 4 (Curry and Webster 1999). Assuming that the difference in velocity between the ice crystals and snowflakes is 1 m/s and that the collection efficiency is unity, the theory (Curry and Webster 1999) simplifies to Equation 3. Using Equations 3-5 (Curry and Webster 1999) we evaluated the ratio of the integrals (Equation 6) and then integrated to produce the final model Equation 7 that can be evaluated with the atmospheric properties detailed further below.

$$(Equation\ 3)\ \frac{dR}{dt} = \frac{\pi}{3}\int_0^R \left(\frac{R+r}{R}\right)^2 n(r)r^3 dr$$

$$(Equation\ 4)\ w_l = \frac{\rho_l}{\rho_a}\int_0^\infty \frac{4\pi}{3} n(r)r^3 dr$$

$$(Equation\ 5)\ \ n(r) = Ar^2 e^{-Br}$$

$$(Equation\ 6)\ \ \frac{dR}{dt}\frac{4\rho_l}{\rho_a w_l} = \frac{\int_0^R \left(\frac{R+r}{R}\right)^2 e^{-Br}\frac{4\pi}{3}r^5 dr}{\int_0^\infty e^{-Br}\frac{4\pi}{3}r^5 dr} = Ratio\ \cong 1.991$$

$$(Equation\ 7)\Delta R = \frac{1.991\rho_a w_l}{4\rho_l}\Delta t$$

The aggregate snow model, with results shown in Figures 6-7, is linear in nature. The key inputs to this model are the mixing ratio of ice and the density of the snowflake. For Equation 6, the following concentration distribution was used. This is an approximation of data collected by Fletcher 1962 and Schemenauer et al 1980.

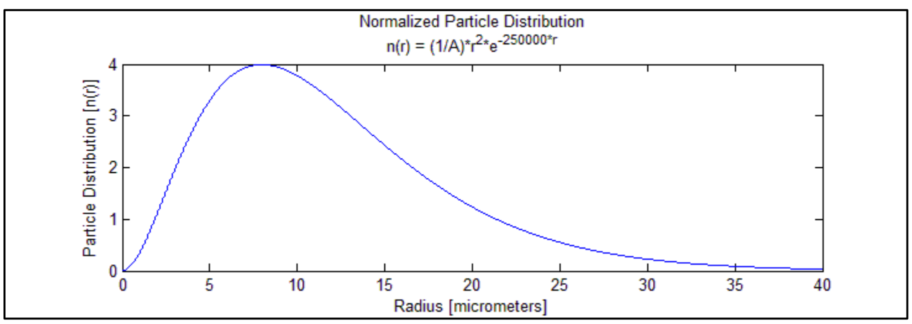

Figure 5. Approximation of the snowflake concentration.

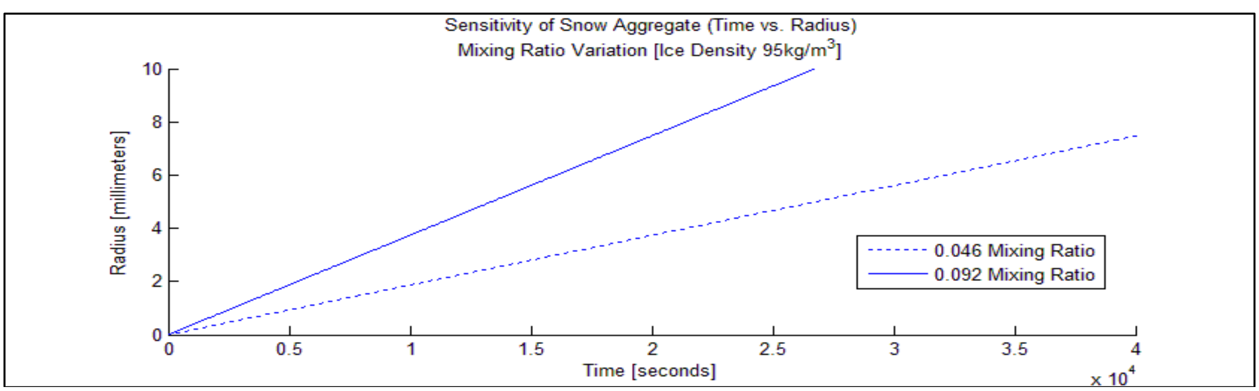

Figure 6. Snowflake growth as a function of time for two ice mixing ratios common to cumulus clouds (parameters from Song, Zhang, and Li 2012).

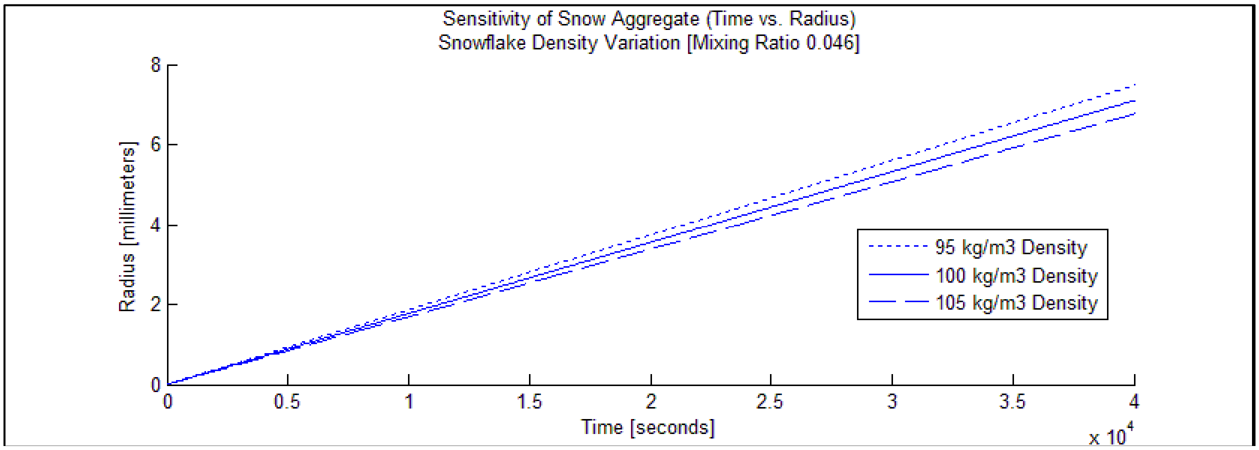

Figure 7. Snowflake growth as a function of time for three different temperatures snowflake densities common to cumulus clouds (parameters from Matrosov et al. 2009).

3. Definition and physical significance of variables

Table 1. Defined variables incorported in modeling.

L_{lv}	Latent Heat of vaporization – Energy absorbed/released during a liquid to vapor phase change
L_{iv}	Latent Heat of sublimation – Energy absorbed or released during a liquid to vapor phase change
S	Supersaturation of water – Ratio of the water vapor pressure to the saturation pressure over water
S_i	Supersaturation of ice – Ratio of water vapor pressure to saturation pressure of water over ice
e_s	Saturation pressure for water over pure liquid
e_{si}	Saturation pressure for water over ice
D_v	Diffusion coefficient for water vapor in air
κ	Atmospheric thermal conductivity coefficient
ρ_l	Density of water
ρ_i	Density of ice
ρ_a	Density of air
R_v	Specific gas constant of water vapor
w_l	Liquid water mixing ratio. – Ratio of the mass of liquid water to the mass of dry air

L_{lv}: Latent heat of vaporization implies large amounts of energy are required to form vapor from liquid water (as in cloud formation) and also limits the height a cloud will travel as heat is released during condensation.

r: In clouds, droplets and ice particles need to have a minimum radius in order to spontaneously grow.

S: Supersaturation increases with elevation because saturation vapor pressure (es) decreases. Thus, conditions for aggregation become more favorable with elevation. Supersaturation begins when the relative humidity reaches 100%, which is required for nucleation to occur.

D_v: Diffusion coefficient influences particle growth rate because adjacent particles take time to travel to the aggregating particle.

Kappa: The ability of a substance to conduct heat. Latent heat liberated by condensation at the drop surface is diffused away from the drop according to kappa. A smaller kappa value inhibits this heat dissipation and slows particle condensation.

w_l: A mass ratio analogous to relative humidity. As w_l increases, there is more water available for diffusional growth, thus conditions for nucleation becomemore favorable.

4. Conclusions and physical insight

Homogeneous nucleation requires very high supersaturation (300% relative humidity; Curry and Webster 1999). Therefore, the particles in the atmosphere that serve as nucleation sites are critical to cloud formation and precipitation. The droplet growth rate curves in Figures 1-6 show that the highest growth rates are inversely proportional to radii. However, the radius of the growing hydrometeor must reach a critical size for the droplet, ice particle, or snowflake to precipitate out of the cloud. Therefore, the larger the radius, the less time to precipitation. In particular, the starting size should be > 70 micrometers to achieve a nucleation-to-precipitation time of less than 30 minutes (Rasmussen 1995). Placing our modeling in the context of rain seeding, if too small of seeds are used, the critical radius (for spontaneous growth) may not be reached, or precipitation may not occur until the cloud has moved away from the intended location of precipitation. Additionally, the model shows that ice sphere growth is approximately 10 times faster than liquid droplet growth. This can be explained when considering that the saturation pressure over ice is much lower than the saturation pressure over water. In general, the fastest growth rates result from higher supersaturation levels, lower temperatures, and larger seed sizes. However, the seed size must be small enough to be maintained by the internal updrafts and turbulence of the clouds so it has time to grow and collect water. The hydrometeor growth sensitivities to temperature and supersaturation indicate that the optimal seed size must be determined considering the measured or expected atmospheric conditions at the time of seeding. Supersaturation and mixing ratio can be thought as the concentration of water present in the given cloud and these factors prove to heavily influence particle growth times for liquid water droplets (supersaturation; Fig. 2) and snowflakes (mixing ratio; Fig. 6). Finally, we acknowledge that these results are influenced by the assumptions made while building the models incorporated. The assumption of constant temperature throughout the cloud and during particle growth likely results in inaccuracies in actual growth rates. However, it probably does not affect the relative growth rates, such as those at different temperatures and the importance of different parameters. With respect to cloud seeding, the model's assumptions would yield inaccurate in calculated times from particle injection to precipitation. But our models, with their assumptions, are effective at demonstrating the importance of cloud seed size e.g. that it must not be too large nor too small in order to produce precipitation at the target locattion.

References

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Appendix

```
% Filename: M3_Liquid
% Author: Michael Kopriva
% Date: 13 November 2014
% SIO 271A Fall 2014
% Project: Precipitation - Altantic Cumulous
%
% M3 Team:
% Maryam Asgari Lamjiri
% Matt Pendergraft
% Michael Koriva
%
% Project Summary: Given a temperature and initial and final precipitation
% sizes, calculate the time required to grow the precipiation.
%
% The corresponding "Thermodynamics of Atmospheres and Oceans" problems are
% 5.5 and 8.2. The associated equations used in this code are 5.26, 5.42,
% 8.2, and 8.4.

    clear all
    Options = {':','-','--','-.','o','x'};
    r0 = 1e-6;
    r2 = 100e-6;
%% %% Saturation Variation
    Tc = -5; %Celsius
    Ssat = [100.5 101 101.5 102]/100;

    figure(100)
    hold on

    for s=1:size(Ssat,2)
        [T_Vec,R_Vec] = Liquid_Calc(Tc,Ssat(s),r0,r2);
        plot(T_Vec,R_Vec*10^6,Options{s})
    end
    legend('0.5% Super Sat','1% Super Sat','1.5% Super Sat','2% Super Sat')
    title({'Sensitivity of Liquid Droplets (Time vs. Radius)';'Supersatuation Variation'})
    ylabel('Radius [micrometers]')
    xlabel('Time [seconds]')
    hold off

    clear Tc Ssat T_Vec R_Vec
%% Temperature Variation
    Tc = [0 -4 -7];
    Ssat = 100.5/100;

    figure(101)
    hold on

    for c=1:size(Tc,2)
```

```
        [T_Vec,R_Vec] = Liquid_Calc(Tc(c),Ssat,r0,r2);
        plot(T_Vec,R_Vec*10^6,Options{c})
    end
    legend('0 Celsius','-4 Celsius','-7 Celsius')
    title({'Sensitivity of Liquid Droplets (Time vs. Radius)';'Temperature Variation'})
    ylabel('Radius [micrometers]')
    xlabel('Time [seconds]')
    hold off


function [Time_Vector,Radius_Vector] = Liquid_Calc(Tc,S,r0,r2)

    Rv = 461.51;%[J/kg-K]
    L_lv = 2274050; % [J/kg] Engineering Toolbox Number
%     L_lv = 2500000;
    Tk = Tc + 273.15; %[K]

    Rho_l = M3_interp(Tc,'rho_l');
    Kappa = M3_interp(Tc,'Kappa'); %[J/m-K-s]
    Dv = M3_interp(Tc,'Dv'); %[m2/s]
    es = M3_interp(Tc,'e_w');

    C = (S-1)/(L_lv^2*Rho_l/(Kappa*Rv*Tk^2)+Rho_l*Rv*Tk/(es*Dv));

    Radius_Vector = r0:(r2-r0)/1000:r2;
    Time_Vector = 1/2*(Radius_Vector.^2-r0^2)/C;

% Filename: M3_Ice

    clear all
    Options = {':','-','--','-.','o','x'};
    r0 = 1e-6;
    r2 = 100e-6;
%% %% Saturation Variation
    Tc = -5;
    Ssat = [100.5 101 101.5 102]/100;

    figure(100)
    hold on

    for s=1:size(Ssat,2)
        [T_Vec,R_Vec] = Ice_Calc(Tc,Ssat(s),r0,r2);
        plot(T_Vec,R_Vec*10^6,Options{s})
    end
    legend('0.5% Super Sat','1% Super Sat','1.5% Super Sat','2% Super Sat')
    title({'Sensitivity of Ice Spheres (Time vs. Radius)';'Supersatuation Variation'})
    ylabel('Radius [micrometers]')
    xlabel('Time [seconds]')
    hold off

    clear Tc Ssat T_Vec R_Vec
```

```matlab
%% Temperature Variation
    Tc = [-1 -8 -14];
    Ssat = 100.5/100;

    figure(101)
    hold on

    for c=1:size(Tc,2)
        [T_Vec,R_Vec] = Ice_Calc(Tc(c),Ssat,r0,r2);
        plot(T_Vec,R_Vec*10^6,Options{c})
    end
    legend('- 1 Celsius',' -8 Celsius',' -14 Celsius')
    title({'Sensitivity of Ice Spheres (Time vs. Radius)';'Temperature Variation'})
    ylabel('Radius [micrometers]')
    xlabel('Time [seconds]')
    hold off


function [Time_Vector,Radius_Vector] = Ice_Calc(Tc,S,r0,r2)

    Rv = 461.51;%[J/kg-K]
    L_iv = 2.83 * 10^6; % [J/kg]
    Tk = Tc + 273.15; %[K]

    Rho_i = M3_interp(Tc,'rho_i');
    Kappa = M3_interp(Tc,'Kappa'); %[J/m-K-s]
    Dv = M3_interp(Tc,'Dv'); %[m2/s]
    es_i = M3_interp(Tc,'e_i');
    es = M3_interp(Tc,'e_w');

    S_i = (S*es)/es_i;
    K_ice = (Rho_i * L_iv^2)/(Kappa * Rv * Tk^2);
    D_ice = (Rho_i * Rv * Tk)/( es_i * Dv);

    time = ((r2^2 - r0^2)*(K_ice + D_ice)/(2*(S_i - 1)));
    time2 = sprintf('%0.3e',time);
    disp(time2)

    Radius_Vector = r0:(r2-r0)/1000:r2;
    Time_Vector = ((Radius_Vector.^2 - r0^2)*(K_ice + D_ice)/(2*(S_i - 1)));


% Filename: M3_Snow
    clear all
    Options = {':','-','--','-.','o','x'};
    r0 = 1e-3;
    r2 = 1e-2;
%% %% Snowflake Density Variation
    wi = 1; %g/kg
    pice = [95 100 105]; %kg/m3
```

```
    figure(100)
    hold on

    for p=1:size(pice,2)
        [T_Vec,R_Vec] = Snow_Calc(wi,pice(p),r0,r2);
        plot(T_Vec*10^3,R_Vec*10^3,Options{p})
    end
    legend('95 kg/m3 Density','100 kg/m3 Density','105 kg/m3 Density')
    title({'Sensitivity of Snow Aggregate (Time vs. Radius)';'Snowflake Density Variation'})
    ylabel('Radius [millimeters]')
    xlabel('Time [seconds]')
    hold off

    clear wi pice T_Vec R_Vec
%% Mixing Ratio Variation
    wi = [0.046 0.092]; %g/kg 0.11
    pice = 100; %Density of Snowflake

    figure(101)
    hold on

    for w=1:size(wi,2)
        [T_Vec,R_Vec] = Snow_Calc(wi(w),pice,r0,r2);
        plot(T_Vec*10^3,R_Vec*10^3,Options{w})
    end
    legend('0.046 Mixing Ratio','0.092 Mixing Ratio')
    title({'Sensitivity of Snow Aggregate (Time vs. Radius)';'Mixing Ratio Variation'})
    ylabel('Radius [millimeters]')
    xlabel('Time [seconds]')
    hold off


function [Time_Vector,Radius_Vector] = Snow_Calc(wi,pice,r0,r2)

    pa = 1.2754; %Density of air
    B = 250000; %Matlab determined constant for n(r) equation
    dx = 0.0001; %size step
    r = 0.001:dx:0.01;
    nr = r.^2.*exp(-B*r);
    Integrand2 = r.^3.*nr; %Equation 8.4 Integral
    Integrand1 = ((r2+r)./r2).^2.*nr.*r.^3; %Equation 8.2 Integral

    Int2 = Integrand2*dx; %Evaluate the integral
    Int1 = Integrand1*dx; %Evaluate the integral

    Ratio = Int1/Int2; %1.9991

    Radius_Vector = r0:(r2-r0)/1000:r2;
    Time_Vector = (Radius_Vector-r0)*4*pice/(pa*wi*Ratio);
```

```
function [varout] = M3_interp(Temp,varin)

  % Description: This code takes a temperature input (double format) and a
    % variable description (string format) and interpolates the deired output
    % value for that temperature. Input limitations are based on available
    % data.
    %
    % The input string options are:
    % 'Kappa' - atmosheric thermal conductivity [J/m-s-K]
    % 'Dv' - water vapor diffusivity [m2/s]
    % 'rho_l' - density of liquid water [kg/m3]
    % 'rho_i' - density of solid water [kg/m3]
    % 'e_w' - saturation pressure of pure liquid water [Pa]
    % 'e_i' - saturation pressure of pure solid water [Pa]
    %
    % Example: Get the density of water at -5 Celsius
    % 999.0963 = interp(-5,'rho_l')

    % Set the data values depending on the variable chosen by the input
    switch varin
        case 'Kappa'
            Data(:,1) = [-40 -30 -20 -10 0 10 20 30];
            Data(:,2) = [2.07 2.16 2.24 2.32 2.40 2.48 2.55 2.63]*10^-2;
        case 'Dv'
            Data(:,1) = [-40 -30 -20 -10 0 10 20 30];
            Data(:,2) = [1.62 1.76 1.91 2.06 2.21 2.36 2.52 2.69]*10^-5;
        case 'rho_l'
            Data(:,1) = [-8     0      10     20     30     40     50     60     70     80     90     100];
            Data(:,2) = [998.65 999.84 999.70 998.21 995.65 992.22 988.04 983.20 977.77 971.80 965.32 958.37];
        case 'rho_i'
            Data(:,1) = [-100   -90    -80    -70    -60    -50    -40    -30    -20    -10     0];
            Data(:,2) = [ 925.70 924.90 924.10 923.30 922.40 921.60 920.80 920.00 919.40 918.90 916.20];
        case 'e_w'
            Data(:,1) = [-50 -45 -40 -35 -30 -25 -20 -15 -10 -5 0 5 10 15 20 25];
            Data(:,2) = [6.35 11.11 18.91 31.38 50.87 80.68 125.38 191.14 286.22 421.42 610.70 8.718 1227.10 1704.2 2337.10 3166.80];
        case 'e_i'
            Data(:,1) = [-40 -35 -30 -25 -20 -15 -10 -5 0];
            Data(:,2) = [12.83 22.32 37.97 63.22 103.20 165.1 259.70 401.4 610.60];
    end

    % Get the appropriate index values in the data
    low = find(Temp>Data(:,1),1,'last');
    high = low+1;

    % Interpolate the data
    slope = (Data(high,2)-Data(low,2))/(Data(high,1)-Data(low,1));
    varout = slope*(Temp-Data(low,1))+Data(low,2);
```