SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2010 Midterm Exam (No calculators, notes, books, PDAs.) **KEY**

Curry and Webster, Ch. 1-4 (and Section12.1)

Here are some numerical values, some of which may be useful on this exam:
Average radius of Earth: 6370 km
Mean reflectivity of the Earth: 0.31
Mean molecular weight of dry air: 29 g/mole
Mean molecular weight of water vapor: 18 g/mole
Gas constant for dry air, R_d: 287 J deg^{-1} kg^{-1}
Gas constant for water vapor, R_v: 461 J deg^{-1} kg^{-1}
Specific heat at constant pressure, c_p: 1004 J deg^{-1} kg^{-1}
Specific heat at constant volume, c_v: 717 J deg^{-1} kg^{-1}
Latent heat of vaporization for water at 273K, L_{lv}: 2.5×10^6 J kg^{-1}
Solar luminosity: 3.92×10^{26} W
Earth-sun distance: 1.50×10^{11} m
Stefan-Boltzmann constant, σ: 5.67×10^{-8} W m^{-2} K^{-4}

1. The albedo provides an important contribution to the radiative balance of the Earth.
   a. What is the definition of the albedo? Albedo is the mean reflectivity of the Earth.
   b. Describe the major components that contribute to the Earth's albedo. The albedo includes, clouds (two-thirds), aerosols, and the surface.
   c. What is the approximate value of the albedo in [%] and [W m^{-2}]? 31% and 114 W m^{-2}.
   d. What techniques have Kiehl and Trenberth (1997) and others used to evaluate this value? Satellite-based measurements of outgoing longwave radiation.
   e. If the albedo increased, how would a simple model predict Earth's temperature would respond? Show your model and how it changes. Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:
   $F_L = \sigma T_{atm}^4$ (assumption 1; Eqn. 3.20)**;** **$F_S = F_L$**(assumption 2-3; Eqn. 3.20)
   $0.25{*}S_0(1-\alpha_p) = \sigma T_{atm}^4$ (Eqn. 3.20, Eqn. 12.)
   As albedo increases, T_{atm} will decrease.

2. Consider the properties of the standard atmosphere, assuming hydrostatic balance and constant lapse rate of Γ=6.5K km^{-1}.
   a. Write the equation for the hydrostatic balance and describe where it comes from.
   b. Derive an expression for the variation of height with pressure $z(p)$, in terms of the surface pressure p_0, surface temperature T_0 and a constant lapse rate Γ.
   c. What are typical values for p_0 and T_0 for modern Earth?
   d. Name one common application of this relationship.

   a. $dp=-\rho g dz$: The hydrostatic balance is the equality of upward (pressure gradient) and downward (gravitational) forces in the atmosphere that results in little net vertical motion.

b. From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{pg}{R_d T}\partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{pg}{R_d T}dz$$

$$\frac{dp}{p} = -\left(\frac{g}{(T_0 - \Gamma z)R_d}\right)dz$$

$$\int_{p_0}^{p}\frac{dp}{p} = \int_0^z\left(-\frac{g}{R_d}\right)\frac{dz}{(T_0 - \Gamma z)}$$

$$\ln\frac{p}{p_0} = \left(\frac{g}{\Gamma R_d}\right)\ln\frac{(T_0 - \Gamma z)}{T_0}$$

$$\left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)} = \left(\frac{T_0 - \Gamma z}{T_0}\right) = \left(1 - \frac{\Gamma z}{T_0}\right)$$

$$z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)}\right]$$

c. $\boldsymbol{p_0}$ =1013 mb; $\boldsymbol{T_0}$= 288K.

d. This relationship is used to determine altitude from pressure measurements on aircraft.

3. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
   a. Thermal equilibrium occurs when two substances have no net exchange of thermal energy.
   b. Relative humidity is the ratio of the vapor pressure of water in the atmosphere to the saturation pressure of water at that temperature, $H=e/e_s$.
   c. "Meteorologists' entropy" is potential temperature and is named this because it is proportional to the entropy of the air parcel.
   d. Saturation pressure partial pressure of gas dissolved in another phase with the most possible dissolved species; OR e.g. the water vapor pressure at equilibrium with pure liquid water phase for a given temperature T.
   e. Stefan-Boltzmann equation says that for black bodies the emitted radiation is proportional to the blackbody temperature to the 4th power, $\mathbf{F = \sigma T_{bb}^4}$ (assumption 1; Eqn. 3.20).

4. Skeptics of global warming often criticize records of global temperature based on the lack of comparable standards and other measurement uncertainties. However, only three fundamental results of physics are required to evaluate the response of mean surface temperature to increasing greenhouse gases. Name them.

   1) The first law of thermodynamics.

2) The Stefan-Boltzman equation (or Planck's law of radiation).
3) The absorption of infrared radiation by greenhouse gases.

5. The saturation vapor pressure (of water) at a temperature of 20°C is 23.4 hPa. Consider moist air at 20°C, a pressure of 1,000 hPa, and a relative humidity of 100%. Find the values:
   a. vapor pressure
   $e=H*e_s = 1*23.4 = 23.4$ hPa [Eqn. 4.34a].
   b. mixing ratio
   $w_v=m_v/m_d=(M_v/M_d)*(e/(p-e))=0.622*(23.4/(1000-23.4))=0.0149$ [Eqn. 4.36].
   c. specific humidity
   $q_v=m_v/(m_v+m_d)=w_v/(w_v+1)=0.0147$ [Eqn. 1.20].
   d. virtual temperature
   $T_v=(1+0.608q_v)*T=(1+0.608q_v)*T=295.6K$ [Eqn. 1.25].

6. Consider air with the same specific humidity as in problem (5), but at a temperature of 30°C. State how you would find the values below, including any laws, equations, and assumptions used, and simplifying as much as possible:
   a. saturation vapor pressure (of water)
   Apply the Clausius-Clapeyron equation, leaving answer in terms of enthalpy of phase change from liquid to vapor.
   $e_2 =e_1*\exp[(-L_{lv}/R_v)*((1/T_2)-(1/T_1))]=(42.4)*\exp[-(2.4x10^6/461)*((1/303)-(1/293))]$
   $=42.1$ hPa.
   b. relative humidity
   Since specific humidity is same as question 5, vapor pressure (e) is also the same as used there: $H=e/e_s = 23.4/42.1 = 56\%$ [Eqn. 4.34a].
   Even if you have not evaluated the exact value, state whether it will increase or decrease relative to the value given in problem (5).
   Since the saturation vapor pressure increases with increased temperature (close to 2x/10 deg C here), the relative humidity will be less than 50% by slightly less than a factor of 2.

7. So-called "mixed-phase" clouds (such as the one shown from Hirst et al., 2001) include liquid water droplets and ice crystals that coexist with water vapor. They have been observed at temperatures of 257-261K and pressures of ~600 mb. In this question, you are asked to apply your knowledge of phase equilibrium to this observation.
   a. Sketch the phase diagram for water, labeling the axes with the range of temperature and pressure relevant to the troposphere and stratosphere.
   b. Label on the phase diagram where water exists as liquid, vapor, and solid simultaneously.
   c. If the only components were water and air, calculate the number of degrees of freedom expected for this system. Be as specific as you can, stating any assumptions and equations that you use. (It is okay at this point to make a simplifying assumption that might be an idealization, just name what it is.)
   d. Consider your answers above. Do the

Fig. 6. Altocumulus castellanus cloud which provided the data shown in Figs. 7–9. The photograph was taken from an altitude of 12,000 ft.

observations of mixed-phase clouds at multiple temperatures at a single altitude violate thermodynamics? Suggest one or more reasons to explain your answer.

a and b. See diagram and labels below.

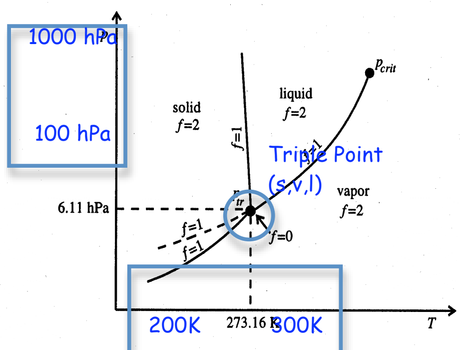

Figure 4.3 *p,T* phase diagram for water. The three curves indicate those points for which two phases coexist at equilibrium. The dashed curve is the extension of the vapor-pressure curve for liquid water to temperatures below 273.16 K. The solid curve below 273.16 K connects the points at which ice and vapor coexist at equilibrium. p_{crit} indicates the pressure and temperature values beyond which liquid water and water vapor are no longer distinguishable from one another. p_{tr} indicates the triple point, the unique *p,T* point at which all three phases coexist.

c. If we assume that the mixed-phase clouds are in thermal, chemical, and mechanical equilibrium, then the Gibbs phase rule applies. Three phases: liquid, solid, vapor. Degrees of freedom by Gibbs phase rule: $f=\chi-\varphi+2=2-3+2=1$. Thus there would be only one temperature that mixed-phase clouds could exist at for each pressure/altitude (T and p form a line).

d. In the real atmosphere, mixed phase clouds are unlikely to be in chemical equilibrium, and the liquid droplets are typically supercooled. If not in equilibrium, then Gibbs' Phase rule does not apply, and the range of temperatures is not limited.