

Lecture Ch. 2b

- Entropy
- Second law of thermodynamics
- Maxwell's equations
- Heat capacity
- "Meteorologist's entropy"

Curry and Webster, Ch. 2 pp. 47-62
Van Ness, Ch. 5-7

Entropy

- Is there a way to quantify "useful" energy?
- Need a measure that is conserved, exact, unique
- While Q is not exact, Q_{rev} is exact
 - Reversible heat is limit of maximum work done
 - Since path is specified, cyclic integral is 0

There exists an additive function of state known as the equilibrium entropy, which can never decrease in a thermally isolated system.

Curry and Webster, Ch. 2 pp. 47-62
Van Ness, Ch. 5-7

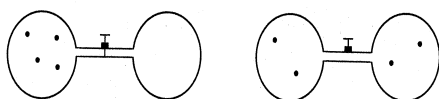


Figure 2.5 Expansion of an ideal gas illustrating the relationship between entropy and probability. Initially, four molecules of the gas are placed in the left bulb, and the right bulb is empty. When the stopcock is opened, the volume doubles, and the molecules are distributed between the left bulb and the right bulb. In this process, the number of possible configurations of molecules, and hence the entropy of the system, has increased.

$$d\eta = c_v d(\ln T) + R d(\ln v) \quad (2.26a)$$

and for isothermal processes

$$\Delta\eta = R \ln\left(\frac{v_2}{v_1}\right) = R \ln\left(\frac{p_1}{p_2}\right)$$

Table 2.1 Ways of arranging four molecules in two bulbs of equal volume.

# in left bulb	# in right bulb	# of ways to achieve configuration, C	Probability of the configuration, P
0	4	1	1/16
1	3	4	4/16
2	2	6	6/16
3	1	4	4/16
4	0	1	1/16
		Total: 16	1

$$\Delta\eta = Nk \ln\left(\frac{V_{fin}}{V_{init}}\right)$$

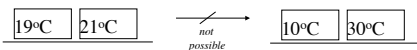
$$\Delta\eta = 4k \ln 2 = k \ln 2^4$$

$$\frac{P_{fin}}{P_{init}} = \frac{C_{fin}}{C_{init}} = 16 = 2^4$$

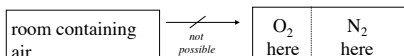
Second law of Thermodynamics



- Heat cannot pass of itself from a colder body to a hotter body.



- A system left to itself cannot move from a less ordered state to a more ordered state.



- The entropy of an isolated system cannot decrease.

$$\Delta S_{system} \geq 0$$

$$\Delta S_{system} = \int_{state 1}^{state 2} \frac{dQ_{rev}}{T_{system}}$$

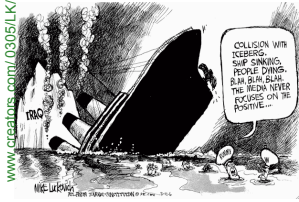
The 2nd Law

- *Energy spontaneously tends to flow only from being concentrated in one place to becoming diffused or dispersed and spread out.*

<http://www.secondlaw.com/two.html>

The 2nd law in practice

- All types of energy spread out



- A sinking ship loses its potential energy to the water it pushes away from itself as it sinks.
- A tire pops. The air never spontaneously fills up the tire.

Clausius' Inequality

$$\Delta\eta_{tot} \geq 0$$

which is known as *Clausius' inequality*. For a reversible process we cannot have $\Delta\eta_{tot} > 0$, since we would have $\Delta\eta_{tot} < 0$ upon reversing the process, which would violate Clausius' inequality. Therefore, $\Delta\eta_{tot} = 0$ for all reversible changes. For the

No process exists in which heat is extracted from a source at a single temperature and converted entirely into useful work, leaving the rest of the world unchanged.

Maxwell's Equations

Since du , dh , da , and $d\eta$ are exact differentials, they obey the Euler condition (2.9). Therefore from (2.31), (2.32), (2.34) and (2.36) we obtain the following set of useful relations called *Maxwell's equations*:

$$\left(\frac{\partial T}{\partial v}\right)_n = -\left(\frac{\partial p}{\partial \eta}\right)_n \quad (2.49)$$

$$\left(\frac{\partial T}{\partial p}\right)_n = \left(\frac{\partial v}{\partial \eta}\right)_p \quad (2.50)$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial \eta}{\partial v}\right)_T \quad (2.51)$$

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial \eta}{\partial p}\right)_T \quad (2.52)$$

Potential Temperature

2.10 Dry Adiabatic Processes in the Atmosphere

In Section 2.4, the following relationship between pressure and temperature was derived for a reversible adiabatic process for an ideal gas:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{R/c_p} \quad (2.61)$$

If we choose $p_0 = 1000$ mb to correspond to a temperature θ , (2.61) becomes

$$\theta = T \left(\frac{p_0}{p}\right)^{R/c_p} \quad (2.62)$$

where R/c_p for dry air is evaluated to be

$$\frac{R}{c_p} = \frac{R}{c_v + R} = \frac{R}{\frac{5}{2}R + R} = \frac{2}{7} = 0.286$$

Virtual Potential Temperature

- Potential Temperature (for moist air)

$$\theta = T \left(\frac{p_0}{p}\right)^{R_d(1-0.26q_v)/c_{pd}} \quad (2.67a)$$

- Virtual Potential Temperature

$$\theta_v = T_v \left(\frac{p_0}{p}\right)^{R_d/c_{pd}} \quad (2.67b)$$

$$\theta_v = T(1 + 0.608q_v) \left(\frac{p_0}{p}\right)^{R_d/c_{pd}}$$

Virtual Temperature

constant. Thus we define a *virtual temperature*, T_v ,

$$T_v = (1 + 0.608q_v) T \quad (1.25)$$

so that the ideal gas law for moist air becomes

$$pv = R_d T_v \quad (1.26)$$

The virtual temperature may be interpreted as the temperature of dry air having the same values of p and v as the moist air under consideration. Since q_v seldom exceeds 0.02, the virtual temperature correction rarely exceeds more than 2 or 3°C; however, it is shown in Chapter 7 that the small virtual temperature correction has an important effect on buoyancy and hence vertical motions in the atmosphere.

$$T_v \geq T, \quad T_v \approx T + [0 \rightarrow 3K]$$

Meteorologists' Entropy

If we choose $p_0 = 1000$ mb to correspond to a temperature θ , (2.61) becomes

$$\theta = T \left(\frac{p_0}{p} \right)^{R/c_p} \quad (2.62)$$

$$\frac{dq}{T} = c_p \frac{dT}{T} - R \frac{dp}{p} = c_p d(\ln T) - R d(\ln p) \quad (2.23)$$

$$d(\ln \theta) = d(\ln T) - \frac{R}{c_p} d(\ln p) \quad (2.63)$$

Comparing (2.63) with (2.23) shows that

$$d\eta = c_p d(\ln \theta) \quad \frac{\eta_2 - \eta_1}{c_p} = \ln \frac{\theta_2}{\theta_1}$$

$$\frac{\theta_2}{\theta_1} = \exp \left[\frac{\eta_2 - \eta_1}{c_p} \right] = \exp \left[\frac{\Delta \eta}{c_p} \right]$$

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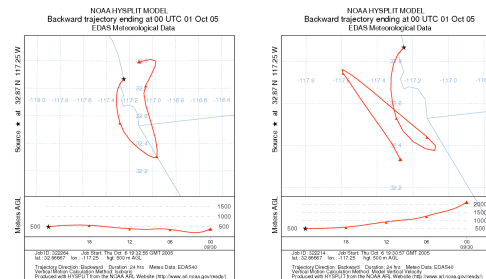
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Example: NOAA HYSPLIT Model

- Trajectories
 - * Single or multiple (space or time) simultaneous trajectories
 - * Optional grid of initial starting locations
 - * Computations forward or backward in time
 - * Default vertical motion using omega field
 - * Other motion options: isentropic, isosigma, isobaric, isopycnic
 - * Trajectory ensemble option using meteorological variations
 - * Output of meteorological variables along a trajectory

<http://www.arl.noaa.gov/ready/hysplit4.html>

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