

# SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2008 Midterm Exam (No calculators, notes, books, PDAs.) [KEY](#)  
Curry and Webster, Ch. 1-4 (and Section 12.1)

Here are some numerical values, some of which may be useful on this exam:

Average radius of Earth: 6370 km

Mean molecular weight of dry air: 29 g/mole

Mean reflectivity of the Earth: 0.31

Mean molecular weight of water vapor: 18 g/mole

Gas constant for dry air:  $287 \text{ J deg}^{-1} \text{ kg}^{-1}$

Gas constant for water vapor:  $461 \text{ J deg}^{-1} \text{ kg}^{-1}$

Specific heat at constant pressure:  $1004 \text{ J deg}^{-1} \text{ kg}^{-1}$

Specific heat at constant volume:  $717 \text{ J deg}^{-1} \text{ kg}^{-1}$

Solar luminosity  $3.92 \times 10^{26} \text{ W}$

Earth-sun distance  $1.50 \times 10^{11} \text{ m}$

Stefan-Boltzmann constant  $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

1. Do you think the average mass of water vapor per unit mass of atmosphere in tropical regions is about the same as in polar regions, or more, or less, and why? State your assumptions and any applicable thermodynamic relationships.

The tropical regions contain much more water vapor than the polar regions, as Fig. 1.1 on page 4 shows dramatically, and the key reason is the strong constraint provided by the Clausius-Clapeyron equation, which expresses the dramatic monotonically increasing dependence of saturation vapor pressure on temperature. Note in Appendix D (page 440) that for a range of typical surface atmospheric temperatures, say from  $-10 \text{ deg C}$  to  $+30 \text{ deg C}$ , the saturation vapor pressure approximately doubles for every  $10 \text{ deg C}$  increase in temperature. In determining the difference between the moisture content of the tropical and polar atmosphere, this strong upper bound on vapor pressure is far more important than variations in relative humidity or other factors that might affect average water vapor concentrations.

2. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
  - a) equivalent temperature  
The temperature that an air parcel would have if all of the water vapor were to condense in an adiabatic isobaric process;  $T_e = T + (L_v/c_{pd}) * w_v$ .
  - b) specific humidity  
The mass fraction of water to air;  $q_v = m_v / (m_v + m_d) = w_v / (w_v + 1)$  [Eqn. 1.20].
  - c) Wien's law  
 $\lambda_{\text{max}} = 2890/T$ ; The maximum wavelength emitted is proportional to the inverse temperature.
  - d) Adiabatic  
A path in which no heat is lost or gained during the process.

e) Carnot cycle

Reversible cyclic process resulting in the maximum possible efficiency; it consists of two isothermal and two adiabatic steps.

f) Work

The quantity of energy transferred from one system to another without an accompanying transfer of entropy; examples include mechanical work ( $dw=F*dx$ ) and expansion work ( $dw=-p*dv$ ).

3. Water exists simultaneously in three phases in the Earth's atmosphere.

a. For an atmosphere consisting only of pure water, what does the coexistence of three phases imply about the temperature and pressure of the system? Be as specific as you can, stating any assumptions and equations that you use.

If we assume that all three phases are in thermal, chemical, and mechanical equilibrium, then the Gibbs phase rule applies. Degrees of freedom by Gibbs phase rule:  $f=\chi-\phi+2=1-3+2=0$ . Thus there is only one temperature and pressure; in addition, the temperature and pressure is the triple point,  $T=273.16K$ ;  $p=6.11$  hPa.

b. If air is also present in the system, how does that change your answer to part (a)? Be as specific as you can, stating any assumptions and equations that you use.

If we assume that all three phases are in thermal, chemical, and mechanical equilibrium, then the Gibbs phase rule applies. Assume we can treat air as one homogeneous mixture, and it is therefore one component. Degrees of freedom by Gibbs phase rule:  $f=\chi-\phi+2=2-3+2=1$ .

There is now one degree of freedom, so a range of temperature and pressure, over which all three phases co-exist.

c. Do the three phases of water in the Earth's atmosphere meet the conditions necessary for Gibbs Phase Rule to apply? State the assumptions and how you know if they are met or not.

In general, the three phases of water in the Earth's atmosphere do not meet the conditions of thermal, chemical, and mechanical equilibrium. There is no thermal equilibrium because there is a vertical gradient in temperature. There is no mechanical equilibrium because pressure varies. Water fraction also varies so there is no chemical equilibrium.

4. Consider the effect on the temperature of the Earth if mirrors were positioned in space to reflect 1% of the solar radiation incident on the Earth (this is a current multi-trillion dollar geo-engineering idea). State and simplify the equations needed to determine the equivalent black-body emission temperature of this planet. State all assumptions and approximations. Solve the equations and identify the values of all constants but you do not need to evaluate the temperature of this planet. Do you expect that this planet will be hotter than the Earth or colder? Discuss the reasons.

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface; (4) no atmosphere. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$$F_L = \sigma T_{\text{surf}}^4 \quad (\text{assumption 1; Eqn. 3.20})$$

$$F_S = F_L \quad (\text{assumption 2-4; Eqn. 3.20})$$

$$0.25 * 0.99 * S_0 (1 - \alpha_p) = 0.25 * 0.99 * S_0 = \sigma T_{\text{surf}}^4 \quad (\text{Eqn. 3.20, Eqn. 12.})$$

$$T_{\text{surf}} = [0.25 * 0.99 * S_0 / \sigma]^{0.25}$$

where  $S_0 = L_0/(4\pi d^2) = 1.3938 \times 10^3 \text{ W m}^{-2}$  (Eqn. 12.),  $\alpha_p=0$ ,  $\sigma=5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

$T_{\text{surf}} = 254.2 \text{ K}$  (previously 254.8K)

The planet is cooler than the Earth with no atmosphere by a very small amount (0.6K) but (If you solve it with a simplified atmosphere, it is much hotter than all of these, but the relative change is still one of cooling!)

5. The saturation vapor pressure (of water) at a temperature of 30°C is 42.4 hPa. Consider moist air at 30°C, a pressure of 1,000 hPa, and a relative humidity of 25%. Find the values:

a. vapor pressure

$$e = H * e_s = 0.25 * 42.4 = 10.6 \text{ hPa [Eqn. 4.34a].}$$

b. mixing ratio

$$w_v = m_v/m_d = (M_v/M_d) * (e/(p-e)) = 0.622 * (10.6/(1000-10.6)) = 0.00666 \text{ [Eqn. 4.36].}$$

c. specific humidity

$$q_v = m_v/(m_v+m_d) = w_v/(w_v+1) = 0.00662 \text{ [Eqn. 1.20].}$$

d. virtual temperature

$$T_v = (1+0.608q_v) * T = (1+0.608q_v) * T = 304.2 \text{ K [Eqn. 1.25].}$$

6. Consider air with the same specific humidity as in problem (5), but at a temperature of 50°C. State how you would find the values below, including any laws, equations, and assumptions used, and simplifying as much as possible:

a. saturation vapor pressure (of water)

Apply the Clausius-Clapeyron equation, leaving answer in terms of enthalpy of phase change from liquid to vapor.

$$e_2 = e_1 * \exp[(-L_v/R_v) * ((1/T_2) - (1/T_1))] = (42.4) * \exp[-(2.4 \times 10^6/461) * ((1/323) - (1/303))] = 123 \text{ hPa.}$$

b. relative humidity

Since specific humidity is same as question 5, vapor pressure (e) is also the same as used there:  $H = e/e_s = 10.6/123 = 8.6\%$  [Eqn. 4.34a].

Even if you have not evaluated the exact value, state whether it will increase or decrease relative to the value given in problem (5).

Since the saturation vapor pressure increases with increased temperature (not 2x/10 deg C here, but a large increase), the relative humidity will be less than 25% by slightly less than a factor of 4.

7. Consider a hydrostatic atmosphere of constant lapse rate of temperature,  $\Gamma$ , such that  $T(z) = T_0 - \Gamma z$  (valid for  $z > 0$ ).

a. Derive a formula for the dependence of pressure  $p$  upon height  $z$ .

From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{p g}{R_d T} \partial z$$

and a constant lapse rate  $\Gamma = -\frac{dT}{dz}$  we get

$$dp = -\frac{pg}{R_d T} dz \left( \frac{-dT/dz}{\Gamma} \right) = -\frac{pg}{\Gamma R_d} \frac{dT}{T}$$

$$\frac{dp}{p} = \left( \frac{g}{\Gamma R_d} \right) \frac{dT}{T}$$

$$\int \frac{dp}{p} = \int \left( \frac{g}{\Gamma R_d} \right) \frac{dT}{T}$$

$$\ln \frac{p}{p_0} = \left( \frac{g}{\Gamma R_d} \right) \ln \frac{T}{T_0}$$

$$p = p_0 \left( \frac{T}{T_0} \right)^{\left( \frac{g}{R_d \Gamma} \right)}$$

which is Eqn 1.48. Then  $p = p_0 [(T_0 - \Gamma z)/T_0]^{(g/\Gamma R_d)}$ .

b. Derive a formula for the dependence of density  $\rho$  upon height  $z$ .

Then dividing both sides by  $RT$  and noting that for an ideal gas  $\rho = \frac{p}{RT}$ , we get

$$\frac{p}{RT} = \rho = \frac{p_0}{RT} \left( \frac{T}{T_0} \right)^{\frac{g}{R_d \Gamma}}$$

Then  $\rho = [p_0/R(T_0 - \Gamma z)] [(T_0 - \Gamma z)/T_0]^{(g/\Gamma R_d)}$ .

c. If  $\Gamma = 10^\circ\text{C}/\text{km}$ , is that higher or lower than the standard atmosphere?

Higher (standard atmosphere is  $\Gamma = 6.5^\circ\text{C}/\text{km}$ ,  $10 > 6.5$ ).