

SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2009 Midterm Exam (No calculators, notes, books, PDAs.) **KEY**
Curry and Webster, Ch. 1-4 (and Section 12.1)

Here are some numerical values, some of which may be useful on this exam:

Average radius of Earth: 6370 km

Mean reflectivity of the Earth: 0.31

Mean molecular weight of dry air: 29 g/mole

Mean molecular weight of water vapor: 18 g/mole

Gas constant for dry air: 287 J deg⁻¹ kg⁻¹

Gas constant for water vapor: 461 J deg⁻¹ kg⁻¹

Specific heat at constant pressure: 1004 J deg⁻¹ kg⁻¹

Specific heat at constant volume: 717 J deg⁻¹ kg⁻¹

Latent heat of vaporization for water at 273K: 2.5×10⁶ J kg⁻¹

Solar luminosity: 3.92×10²⁶ W

Earth-sun distance: 1.50×10¹¹ m

Stefan-Boltzmann constant: 5.67×10⁻⁸ W m⁻² K⁻⁴

1. What is the hydrostatic balance? Why is it useful in describing the atmosphere? Describe the dependence of atmospheric pressure (p) on altitude (z), explicitly (i.e. p=f(z)) for a region of the lower atmosphere in which the rate of decrease of temperature (T) with increasing altitude was approximately constant. State your assumptions and any applicable thermodynamic relationships.

The hydrostatic balance is the equality of upward (pressure gradient) and downward (gravitational) forces in the atmosphere that results in little net vertical motion.

It is useful in describing the atmosphere because it is a reasonable approximation for many stable atmospheric conditions.

From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{pg}{R_d T} \partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{pg}{R_d T} dz \left(\frac{-dT/dz}{\Gamma} \right) = -\frac{pg}{\Gamma R_d} \frac{dT}{T}$$

$$\frac{dp}{p} = \left(\frac{g}{\Gamma R_d} \right) \frac{dT}{T}$$

$$\int \frac{dp}{p} = \int \left(\frac{g}{\Gamma R_d} \right) \frac{dT}{T}$$

$$\ln \frac{p}{p_0} = \left(\frac{g}{\Gamma R_d} \right) \ln \frac{T}{T_0}$$

$$p = p_0 \left(\frac{T}{T_0} \right)^{\left(\frac{g}{R_d \Gamma} \right)}$$

which is Eqn 1.48. Then $p = p_0 [(T_0 - \Gamma z) / T_0]^{(g/\Gamma R_d)}$.

2. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:

a) virtual temperature

The temperature of dry air having the same values of p and v as the moist air under consideration: $T_v = (1 + 0.608q_v) * T$ [Eqn. 1.25].

g) relative humidity

The amount of water vapor present in the atmosphere normalized by the amount present at saturation $H = e/e_s$ [Eqn. 4.34a].

h) equilibrium

Equilibrium occurs when two substances have no net exchange of mechanical, chemical, or thermal energy.

i) Reversible

A reversible process is one which occurs in an infinite number of infinitesimally small steps, which at any point in time can be nudged to go in the opposite direction.

j) saturation pressure

partial pressure of gas dissolved in another phase with the most possible dissolved species; OR e.g. the water vapor pressure at equilibrium with pure liquid water phase for a given temperature T

k) Planck's equation (of radiation)

A small sketch of irradiance versus wavelength for a blackbody of a specified temperature (as in fig. 3.1) is a very good and complete answer here. Providing Eqn. 3.19

$$F_{\lambda}^* = \frac{2\pi hc^2}{\lambda^5 \left[\exp\left(\frac{hc}{k\lambda T}\right) - 1 \right]}$$

(or a simplified form thereof) is also sufficient.

3. Skeptics of global warming often criticize detailed global climate models (run on large computers) because of “uncertainties in the boundary conditions” that result in unpredictable responses in the nonlinear equations that describe atmospheric circulation. Would this criticism apply to a simplified climate model? State and simplify the equations needed to determine the equivalent black-body emission temperature of the Earth as a proxy for climate, including a greenhouse effect. State all assumptions and approximations. Identify the values of all constants but you do not need to evaluate the temperature. How is this calculation affected by “uncertainties in the boundary conditions”? What would be an appropriate response to the type of criticism stated above on the basis of this simplified climate model?

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface; (4) atmosphere completely absorbs infrared radiation. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$$F_L = \sigma T_{\text{atm}}^4 \quad (\text{assumption 1; Eqn. 3.20})$$

$$F_s = F_L \quad (\text{assumption 2-3; Eqn. 3.20})$$

$$0.25 * S_0 (1 - \alpha_p) = \sigma T_{\text{atm}}^4 \quad (\text{Eqn. 3.20, Eqn. 12.})$$

$$T_{\text{atm}} = 255\text{K}$$

where $S_0 = L_0 / (4\pi d^2) = 1.3938 \times 10^3 \text{ W m}^{-2}$ (Eqn. 12.), $\alpha_p = 0$, $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

$$F_{\text{surf}} = 2F_{\text{atm}} \quad (\text{assumption 4})$$

$$\sigma T_{\text{surf}}^4 = 2\sigma T_{\text{atm}}^4$$

$$T_{\text{surf}} = 303\text{K}$$

The planet is warmer than observed (288K) with an atmosphere that is perfectly absorbing in the infrared.

This simplified climate model does not include the non-linear feedbacks of a circulation model because it is an equilibrium model, thus it is not subject to the types of boundary conditions of concern to skeptics.

While this model cannot predict specific changes to the distribution of temperature (and precipitation) in the Earth system which are subject to uncertainties in boundary conditions, the basic principle that more IR absorption in the atmosphere must lead to a higher equivalent blackbody temperature of the Earth will still hold. In other words, the extra energy has to go somewhere.

4. In the springtime Arctic, “mixed-phase” clouds frequently dominate the skies. In these clouds, liquid water droplets and ice crystals coexist, making them a mixture of “warm” and “cold” clouds.
- b) If the only components were water and air, identify the number of phases present and the number of degrees of freedom. Be as specific as you can, stating any assumptions and equations that you use. (It is okay at this point to make a simplifying assumption that might be an idealization; you might want to read part (d) before doing (a).)

If we assume that the mixed-phase clouds are in thermal, chemical, and mechanical equilibrium, then the Gibbs phase rule applies. Three phases: liquid, solid, vapor. Degrees of freedom by Gibbs phase rule: $f = \chi - \phi + 2 = 2 - 3 + 2 = 1$. Thus there is a unique temperature for each pressure (T and p form a line).

c) Now consider that there are also particles in the air; how does that change your answer to part (a)? Be as specific as you can, stating any assumptions and equations that you use.

If we assume that the mixed-phase clouds are in thermal, chemical, and mechanical equilibrium, then the Gibbs phase rule applies. Three phases: liquid, solid, vapor. Degrees of freedom by Gibbs phase rule: $f = \chi - \phi + 2 = 3 - 3 + 2 = 2$. Thus there is a range of temperatures for each pressure (T and p form a plane).

d) Given your assumptions in parts (a) and (b), how limited would be the range of temperatures you would expect to see mixed-phase clouds at 900 hPa?

For (a): very limited, i.e. one temperature. For (b) less limited, there would be a range of temperatures.

e) In the real atmosphere, do you think that a mixed-phase cloud system would be in chemical equilibrium? If the clouds are not in chemical equilibrium, how would this affect the equations you used in part (a)? State the assumptions and how you know if they are met or not.

In the real atmosphere, mixed phase clouds are unlikely to be in chemical equilibrium. If not in equilibrium, then Gibbs' Phase rule does not apply, and the range of temperatures is not limited. We expect this range of behavior to be likely, given the variability of conditions in which mixed phase clouds "frequently" exist. (However, in many cases they have been shown to be "close" to equilibrium.)

5. The saturation vapor pressure (of water) at a temperature of 20°C is 23.4 hPa. Consider moist air at 20°C, a pressure of 1,000 hPa, and a relative humidity of 50%. Find the values:

a. vapor pressure

$$e = H * e_s = 0.5 * 23.4 = 11.7 \text{ hPa [Eqn. 4.34a].}$$

b. mixing ratio

$$w_v = m_v / m_d = (M_v / M_d) * (e / (p - e)) = 0.622 * (11.7 / (1000 - 11.7)) = 0.0074 \text{ [Eqn. 4.36].}$$

c. specific humidity

$$q_v = m_v / (m_v + m_d) = w_v / (w_v + 1) = 0.0073 \text{ [Eqn. 1.20].}$$

d. virtual temperature

$$T_v = (1 + 0.608 q_v) * T = (1 + 0.608 q_v) * T = 294.3 \text{ K [Eqn. 1.25].}$$

6. Consider air with the same specific humidity as in problem (5), but at a temperature of 30°C. State how you would find the values below, including any laws, equations, and assumptions used, and simplifying as much as possible:

a. saturation vapor pressure (of water)

Apply the Clausius-Clapeyron equation, leaving answer in terms of enthalpy of phase change from liquid to vapor.

$$e_2 = e_1 * \exp[(-L_v / R_v) * ((1/T_2) - (1/T_1))] = (42.4) * \exp[-(2.4 \times 10^6 / 461) * ((1/303) - (1/293))] = 42.1 \text{ hPa.}$$

b. relative humidity

Since specific humidity is same as question 5, vapor pressure (e) is also the same as used there: $H = e / e_s = 11.7 / 42.1 = 28\%$ [Eqn. 4.34a].

Even if you have not evaluated the exact value, state whether it will increase or decrease relative to the value given in problem (5).

Since the saturation vapor pressure increases with increased temperature (close to $2x/10$ deg C here), the relative humidity will be less than 50% by slightly less than a factor of 2.

7. Latent heat constitutes a small but significant fraction of the atmospheric radiation balance. Consider moist air near the Earth's surface with specific humidity of 0.015.
- a. Calculate the heat required for a phase change of the water in 1 kg of moist air from liquid to vapor at 273K.

Assume a constant pressure process since "near Earth's surface" (i.e. $p=1$ atm).

Assume a constant temperature phase change at 273K.

$$Q=\Delta H=q_v L=(0.015)(2.5 \times 10^6) \text{ J kg}^{-1}=38 \text{ kJ kg}^{-1}$$

- b. Calculate the heat required for warming 1 kg moist air in the lower atmosphere from 273K to 293K.

Assume a constant pressure process since "near Earth's surface" (i.e. $p=1$ atm).

Approximate moist air has heat capacity of dry air.

$$Q=\Delta H=c_p \Delta T=(1004 \text{ J deg}^{-1} \text{ kg}^{-1})(293-273 \text{ K})=20 \text{ kJ kg}^{-1}$$

- c. Estimate the relative values of parts (b) and (c) to explain why latent heat accounts for a substantial fraction of energy in the Earth's atmosphere.

Even though water vapor is a very small fraction of each kg of air, the latent heat required for a phase change is almost twice that needed to warm a kg of air for 20K.