SIO 217a Atmospheric and Climate Sciences I: Atmospheric Thermodynamics

Fall 2011 Midterm Exam (No calculators, notes, books, PDAs.) **KEY**

Curry and Webster, Ch. 1-4 (and Section12.1)

Here are some numerical values, some of which may be useful on this exam:
Average radius of Earth: 6370 km
Mean reflectivity of the Earth: 0.31
Mean molecular weight of dry air: 29 g/mole
Mean molecular weight of water vapor: 18 g/mole
Gas constant for dry air, R_d: 287 J deg^{-1} kg^{-1}
Gas constant for water vapor, R_v: 461 J deg^{-1} kg^{-1}
Specific heat at constant pressure, c_p: 1004 J deg^{-1} kg^{-1}
Specific heat at constant volume, c_v: 717 J deg^{-1} kg^{-1}
Latent heat of vaporization for water at 273K, L_{lv}: 2.5×10^6 J kg^{-1}
Solar luminosity: 3.92×10^{26} W
Earth-sun distance: 1.50×10^{11} m
Stefan-Boltzmann constant, σ: 5.67×10^{-8} W m^{-2} K^{-4}

1. Almost one-third of the Earth's incoming solar radiation is reflected back to space.
   a. Name the property of the Earth controls the fraction of incoming light reflected. Albedo is the mean reflectivity of the Earth.
   b. Calculate the amount of incoming solar radiation at the top of the atmosphere [in W m^{-2}]. Instantaneous at solar noon =(luminosity)/(4pi*ESdistance^2)
      $=(3.92\times10^{26})/(4*3.14*(1.50\times10^{11})^2)=1390$ W m^{-2};
      Averaged over Earth surface = (1390 W m^{-2})/4=342 W m^{-2}.
   c. What happens to the energy from the incoming radiation that is not reflected? The remaining energy is absorbed by the Earth and the atmosphere and then re-emitted.
   d. If the amount of light reflected were *decreased*, how would a simple model *with no greenhouse effect* predict Earth's temperature would respond? Give your model and state its assumptions. Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:
      $\mathbf{F_L = \sigma T_{atm}^4}$ (assumption 1; Eqn. 3.20); $\mathbf{F_S = F_L}$(assumption 2-3; Eqn. 3.20)
      $\mathbf{0.25*S_0(1-\alpha_p) = \sigma T_{atm}^4}$ (Eqn. 3.20, Eqn. 12.)
      As albedo decreases, $\mathbf{T_{atm}}$ will increase.

2. Consider the properties of the standard atmosphere, assuming hydrostatic balance and constant lapse rate of Γ=6.5K km^{-1}.
   a. Write the equation for the hydrostatic balance and describe where it comes from.
   b. Derive an expression for the variation of height with pressure $z(p)$, in terms of the surface pressure p_0, surface temperature T_0 and a constant lapse rate Γ.
   c. What are typical values for p_0 and T_0 for modern Earth?

d. Name one common application of this relationship.

a. ***dp=-ρgdz***: The hydrostatic balance is the equality of upward (pressure gradient) and downward (gravitational) forces in the atmosphere that results in little net vertical motion.

b. From the hydrostatic equation for an ideal gas (Eqn. 1.42)

$$\partial p = -\frac{pg}{R_d T}\partial z$$

and a constant lapse rate $\Gamma = -\frac{dT}{dz}$ we get

$$dp = -\frac{pg}{R_d T}dz$$

$$\frac{dp}{p} = -\left(\frac{g}{(T_0 - \Gamma z)R_d}\right)dz$$

$$\int_{p_0}^{p}\frac{dp}{p} = \int_0^z\left(-\frac{g}{R_d}\right)\frac{dz}{(T_0 - \Gamma z)}$$

$$\ln\frac{p}{p_0} = \left(\frac{g}{\Gamma R_d}\right)\ln\frac{(T_0 - \Gamma z)}{T_0}$$

$$\left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)} = \left(\frac{T_0 - \Gamma z}{T_0}\right) = \left(1 - \frac{\Gamma z}{T_0}\right)$$

$$z = \frac{T_0}{\Gamma}\left[1 - \left(\frac{p}{p_0}\right)^{\left(R_d\Gamma/g\right)}\right]$$

c. p_0 =1013 mb; T_0= 288K.

d. This relationship is used to determine altitude from pressure measurements on aircraft.

3. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
   a. mechanical equilibrium occurs when two substances have no net exchange of force, i.e. $p_1=p_2$.
   b. adiabatic means there is no exchange of heat into or out of the system; A path in which no heat is lost or gained during the process (Q=0).
   c. virtual potential temperature is the temperature a parcel would have if there were no water vapor in it and if it were brought adiabatically and reversibly to p_0 (usually 1 atm).

$$\boxed{\theta_v = T(1 + 0.608q_v)\left(\frac{p_0}{p}\right)^{R_d/c_{pd}}}$$

   d. Wien's law λ_{max}=2890/T; The maximum wavelength emitted is proportional to the inverse temperature.
   e. exact differential is a function ξ for which dξ has the properties (1) for any closed path $\oint d\xi = 0$, and (2) for ξ(x,y) where x and y are independent, then

$$d\xi = \left(\frac{\partial\xi}{\partial x}\right)_y dx + \left(\frac{\partial\xi}{\partial y}\right)_x dy \equiv Mdx + Ndy \Rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y .$$

f. state variable is a path-independent function ξ for which $d\xi$ has the properties (1) for any closed path $\oint d\xi = 0$, and (2) for $\xi(x,y)$ where x and y are independent, then $d\xi = \left(\frac{\partial \xi}{\partial x}\right)_y dx + \left(\frac{\partial \xi}{\partial y}\right)_x dy \equiv Mdx + Ndy \Rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$.

g. Gibbs' phase rule says that for a system in thermal, chemical, and mechanical equilibrium, then the number of degrees of freedom are given by the Gibbs phase rule: $f=\chi-\varphi+2$ (for χ components in φ phases).

4. Skeptics of global warming often criticize records of global temperature based on the lack of comparable standards and other measurement uncertainties. However, only three fundamental results of physics are required to evaluate the response of mean surface temperature to increasing greenhouse gases. Name them.
1) The first law of thermodynamics.
2) The Stefan-Boltzman equation (or Planck's law of radiation).
3) The absorption of infrared radiation by greenhouse gases.

5. The saturation vapor pressure (of water) at a temperature of 30°C is 42.4 hPa. Consider moist air at 30°C, a pressure of 1000 hPa, and a relative humidity of 25%. Find the values:
a. vapor pressure
$e=H*e_s = 0.25*42.4 = 10.6$ hPa [Eqn. 4.34a].
b. mixing ratio
$w_v=m_v/m_d=(M_v/M_d)*(e/(p-e))=0.622*(10.6/(1000-10.6))=0.00666$ [Eqn. 4.36].
c. specific humidity
$q_v=m_v/(m_v+m_d)=w_v/(w_v+1)=0.00662$ [Eqn. 1.20].
d. virtual temperature
$T_v=(1+0.608q_v)*T=(1+0.608q_v)*T=304.2K$ [Eqn. 1.25].

6. The saturation vapor pressure (of water) doubles approximately every 10°C in typical atmospheric conditions. This *strong dependence* of saturation vapor pressure on temperature (i.e. this large change of doubled pressure per 10°C increase in temperature) provides many of the unique cloud feedbacks that govern climate on Earth.

(a) Give *the differential form* of the equation that describes this increase in saturation vapor pressure with temperature. This equation is the Clausius-Clapeyron equation, and the differential form is given as:

$$\boxed{\left(\frac{\partial p}{\partial T}\right)_g = \frac{L_{lv} p}{R_v T^2}}$$

(b) State the laws and assumptions used to derive the equation in (a). The laws and assumptions required are: (1) Definition of free energy and its properties as a state function; (2) Constant free energy of phase changes at constant temperature and pressure; (3) phase change from liquid to gas such that $v_L << v_V$; (4) ideal gas law for vapor, $v_V = R_V T/p$.

(c) Identify the water property *that appears in equation (a)* that makes this dependence strong. (Hint: The answer is a quantity related to water that is *not saturation vapor pressure*.) Latent heat, L_{lv}, which is quite high for water.

(d) Describe the characteristic of the *water molecule* that makes the property in (c) so unique. The high polarity of the water molecule (which results in hydrogen bonding) is one important property that makes its latent heat so high.

7. Warm clouds (such as those found frequently in La Jolla) include liquid water droplets and water vapor. Typical temperatures are 20°C. In this question, you are asked to apply your knowledge of phase equilibrium to the behavior of water shown in the diagram below.

a. Using the diagram, label the region of temperature and pressure where liquid and vapor coexist for 20°C, *at equilibrium*, assuming pure water. (Hint: RH~100%, e_s=23 hPa at T=293K.) **Label this region A.** See diagram below.

b. If water condenses releasing latent heat *at constant pressure*, what equation would determine the new location on the diagram? Indicate the direction of change from A by **an arrow labeled B**. $w_sL_{lv}=c_p\Delta T$, temperature will increase; see diagram below.

c. What would happen to the cloud droplets (i.e. the liquid water) at B? They would tend to evaporate.

d. Does this happen to clouds? Why or why not? This does not happen in clouds because in the atmosphere, clouds are unlikely to be in chemical equilibrium. If not in equilibrium, then Gibbs' Phase rule does not apply, and the range of temperatures is not limited.

e. Now consider an ice cloud with solid and vapor water present; **label this region C** where an ice cloud could be at equilibrium in a pure water system. See diagram below.

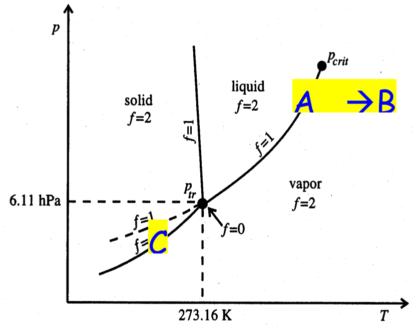

Figure 4.3 *p,T* phase diagram for water. The three curves indicate those points for which two phases coexist at equilibrium. The dashed curve is the extension of the vapor-pressure curve for liquid water to temperatures below 273.16 K. The solid curve below 273.16 K connects the points at which ice and vapor coexist at equilibrium. p_{crit} indicates the pressure and temperature values beyond which liquid water and water vapor are no longer distinguishable from one another. p_{tr} indicates the triple point, the unique *p,T* point at which all three phases coexist.