SIO 217A Midterm Exam Key Fall 2007

Curry and Webster, Ch. 1-4 (and Section12.1)

1. Answer **both** part (a) and part (b).
   a) If air has a pressure of 1026.8 hPa and a mixing ratio of 0.005, calculate vapor pressure.
   w_v=0.622*e/(p-e); 8.19 mb
   b) air has a specific humidity of 0.0196 and a temperature of 30°C, calculate potential temperature.
   $\Theta = T*(p_0/p)^{R/cp}$ =300.9 K[Eqn. 2.67b]

2. Sketch the pressure-temperature phase diagram for water. On the graph, label
   a) all phases and their transitions,
   b) the triple point,
   c) the temperature and pressure at which water boils at sea level,
   d) the location of supercooled water,
   e) the location of supersaturated water vapor.

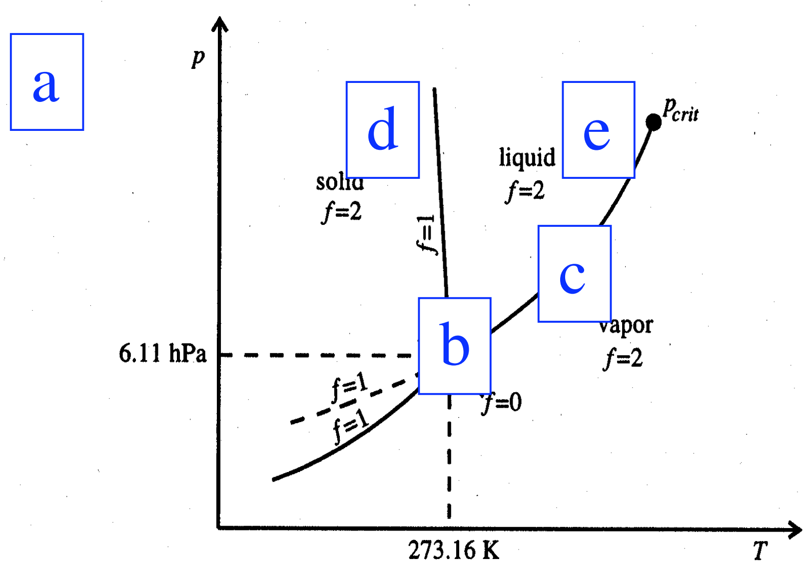


Figure 4.3 *p,T* phase diagram for water. The three curves indicate those points for which two phases coexist at equilibrium. The dashed curve is the extension of the vapor-pressure curve for liquid water to temperatures below 273.16 K. The solid curve below 273.16 K connects the points at which ice and vapor coexist at equilibrium. p_{crit} indicates the pressure and temperature values beyond which liquid water and water vapor are no longer distinguishable from one another. p_{tr} indicates the triple point, the unique *p,T* point at which all three phases coexist.

   f)

3. Consider a planet that is identical to Earth in all respects except for its albedo. State and simplify the equations needed to determine the equivalent black-body emission temperature of this planet if α_p=0. State all assumptions and approximations. Solve the equations but you do not need to evaluate the temperature of this planet. Do you expect that this planet will be hotter than the Earth or colder? Discuss the reasons.

Assume that: (1) the earth behaves as a blackbody, (2) atmosphere is transparent to non-reflected portion of the solar beam; (3) atmosphere in radiative equilibrium with surface; (4) no atmosphere. Then, at equilibrium, the incoming shortwave flux and outgoing longwave flux are equal (i.e. there is no accumulation) so for the normal solar luminosity we can write:

$\mathbf{F_L = \sigma T_{surf}^4}$ (assumption 1; Eqn. 3.20)

$\mathbf{F_S = F_L}$(assumption 2-4; Eqn. 3.20)

$\mathbf{0.25*S_0(1- \alpha_p) = 0.25*S_0 = \sigma T_{surf}^4}$ (Eqn. 3.20, Eqn. 12.)

$\mathbf{T_{surf} = [0.25*S_0/\sigma]^{0.25}}$

where $S_0 = L_0/(4\pi d^2) = 1.3938 \times 10^3$ W m^{-2} (Eqn. 12.), $\alpha_p=0$, $\sigma=5.67\times10^{-8}$ W m^{-2} K^{-4}

$\mathbf{T_{surf} = 280}$ **K**

The planet is hotter than the Earth with no atmosphere (255K) but it is not as hot as the Earth with a simplified atmosphere (303K) or the current observed temperature (288K) because it has no greenhouse effect. (If you solve it with a simplified atmosphere, it is much hotter than all of these!)

4. Draw a diagram illustrating the annual mean energy balance of the Earth. Include the major pathways for energy transfer by both solar and terrestrial radiation.

Fig. 12.2 on page 334 is the diagram for Earth that was sought, although Fig. 14.8 or equivalent was discussed in class (but not assigned in the text) and is a possible partial answer too as a general illustration of the radiative transfer pathways for an idealized planet.

a) In a paragraph (50 words maximum) describe the most important features of this diagram and how they affect the Earth's climate.

The last paragraph on p. 333 is a good discussion. There was no need to memorize the numerical values of the various energy transfer pathways, but a qualitative understanding was expected of the relative magnitudes of the major features.

b) In another paragraph (50 words maximum) describe how this diagram might change as human activities increase the atmospheric concentration of carbon dioxide.

See the Lindzen and Emanuel article, particularly the sections on "response of the greenhouse effect to increasing concentrations of trace gases," and "Greenhouse feedbacks." The essential concept is that adding carbon dioxide increases the infrared opacity of the atmosphere, raises the mean effective emitting altitude, and thus leads to a warming in order to maintain a radiative equilibrium between the energy absorbed from the Sun and that emitted by the Earth system. Feedbacks then come into play, of which one of the most important is the water vapor feedback. Several other possibly important feedbacks, such as those relating to clouds, are imperfectly understood.

5. Draw 3 graphs showing how the average values of the following 3 quantities vary with altitude in the troposphere: temperature, density, pressure. Explain the physical reasons that determine the important features of these curves.

See Figs. 1.7, 1.5, and 1.3 and accompanying discussion in the text. The key physical reasons include the compressibility of the atmosphere (as opposed to the ocean), so that density and pressure both decrease approximately exponentially with height in a hydrostatic atmosphere, and the fact that the troposphere is mainly heated from below, so that temperature decreases with height at a rate that is qualitatively approximated by the dry adiabatic lapse rate. Note that only discussions of tropospheric variations were sought in the question, so there was no need to explain the existence of the stratosphere, etc.

6. Define the following terms in 10 words or less; an equation, graph, or sketch may be added if appropriate:
   a) saturation vapor pressure
   partial pressure of gas dissolved in another phase with the most possible dissolved species; OR e.g. the water vapor pressure at equilibrium with pure liquid water phase for a given temperature T
   b) virtual potential temperature
   $\Theta_v = T_v*(p_0/p)^{R/cp}$ [Eqn. 2.67b]
   c) Gibbs phase rule
   $f=\chi-\varphi+2$, the number of degrees of freedom in a system having thermal, mechanical and chemical equilibrium.
   d) ideal gas
   vapor whose molecules have collisions with perfect elasticity, typical at low pressures (for air ≤1 atm) and high temperatures (typically ≥300K); OR vapor that satisfies pv=RT and has the properties that $dh=c_p dT$, $du=c_v dT$, $c_p-c_v=R$ (p. 44).
   e) reversible
   process which is carried out at series of infinitessimally small mass-conserving steps, for which the process could at any point proceed in the opposite direction.
   f) state function
   function ξ for which dξ has the properties (1) for any closed path $\oint d\xi = 0$, and (2) for ξ(x,y) where x and y are independent, then.

$$d\xi = \left(\frac{\partial \xi}{\partial x}\right)_y dx + \left(\frac{\partial \xi}{\partial y}\right)_x dy \equiv Mdx + Ndy \Rightarrow \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

7. Consider a hydrostatic atmosphere where pressure varies with height in the following way: $p(z) = \dfrac{p_0}{1+\left(\dfrac{z}{H}\right)^2}$
   a. Determine the corresponding variation of temperature with height (valid for z>0).

1) Separate variables.
2) Take the derivative (of both sides) with respect to z.
3) Group dp/dz on left, and p(z) terms on right.
4) Use hydrostatic eqn (dp/dz=-rho*g) to replace dp/dz on left.
5) Use ideal gas p(z)/rho=RT(z) to remove rho and p(z).
6) Solve for T(z).

$$p(z) = \frac{p_0}{1 + \left(\frac{z}{H}\right)^2}$$

Try using the steps given above to do this part for homework (extra credit will be given).

Ask me questions if you get stuck!

$$T(z) = \frac{gH^2}{2zR}\left[1 + \left(\frac{z}{H}\right)^2\right]$$

c. Use the result from (a) and H for a homogeneous atmosphere to evaluate the height at which Γ=10ºC km^{-1}.

$$T(z) = \frac{gH^2}{2zR}\left[1 + \left(\frac{z}{H}\right)^2\right]$$

$$\frac{dT}{dz} = -\frac{gH^2}{2z^2R} + \frac{g}{2R} = -0.01 \text{ K/m}$$

$$-0.02Rz^2 = g\left(z^2 - H^2\right)$$

$$gH^2 = (g + 0.02R)z^2$$

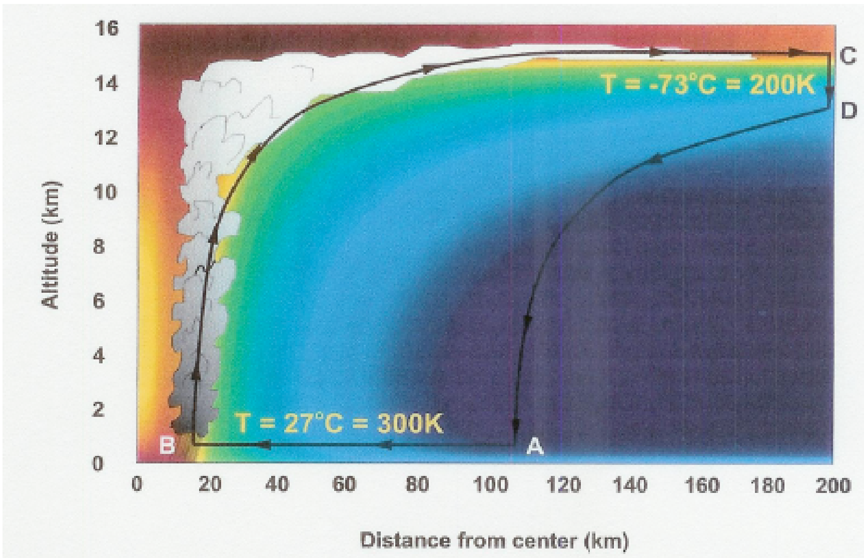
$$z = H\sqrt{\frac{g}{g + 0.02R}} = 8\sqrt{\frac{9.8}{9.8 + 0.02(287)}} = 6.35 \text{ km}$$

where we have used H=8 km for a homogeneous atmosphere [p. 28]

8. *Consider the energy cycle of the mature hurricane shown below (from Emmanuel, 2005). Air spirals inward close to the sea surface, between points A and B, acquiring heat from the ocean by evaporation of seawater at approximately constant 27ºC. Air then ascends in the eyewall, from B to C, without acquiring or losing heat other than that produced when water vapor condenses. Between C and D at approximately -*

73ºC, the air loses the heat it originally acquired from the ocean. Finally, between D and A, the air returns to its starting point.

Calculate the maximum efficiency of work done by heat transferred in this energy cycle. State all assumptions and approximations. Is this process reversible? Why or why not?



Maximum efficiency is for reversible adiabatic steps for B-C and D-A, and isothermal steps for A-B and C-D, also known as a Carnot cycle.

Tcold=200K Earth's temperature

Thot=300K Sun's temperature

The maximum efficiency of this Carnot cycle is given by

$$\eta = 1 - \frac{T_{Cold}}{T_{Hot}}$$

Maximum efficiency=0.33

The process is not reversible because the system is not closed because of loss of water by rain.