Growth of Cloud Droplets by Condensation

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1 Introduction

Cloud droplet growth is an integral part of atmospheric physics, and understanding it is vital for modeling cloud formation, parcel motion, and precipitation. In the atmosphere, the initial formation of cloud droplets occurs due to water vapor condensing onto cloud condensation nuclei (CCN), which are particles such as aerosols that are present in the atmosphere. Due to the Kelvin Effect, this condensation only occurs when the air contains slightly more water vapor than it normally holds for a given temperature, i.e. the air is supersaturated. Thus, it is important to understand what conditions of atmospheric supersaturation and CCN promote cloud droplet nucleation and growth. Kohler curves describe absolute supersaturation levels at which cloud droplets become nucleated and grow stably. However, these curves do not describe the evolution of the supersaturation that occurs during uplifting of air parcels during cloud formation. How supersaturation and droplet growth are affected by adiabatic cooling due to uplift are described by a set of equations which are not easily solved analytically. In this paper we present results by numerically solving these equations. In Section 2, we present the governing equations. We next describe the numeric method used to solve these equations in Section 3. Lastly, in Section 4 we state our results and draw some general conclusions.

2 Background

Initial cloud droplet growth occurs by diffusing water vapor onto the drop under conditions in which water vapor pressure from the environment is greater than the saturation vapor pressure of the drop. During this phase change from vapor to liquid, latent heat is released and warms the drop. This leads to an increase in the saturation vapor pressure of the drop, which in turn reduces the vapor pressure gradient necessary for diffusion. This reduces the drop growth rate and forces heat to diffuse from the drop to the environment. By assuming that the only process in which the drop can increase it's mass is through diffusion, Mason (1971) found the growth rate of the drop to be

$$r\frac{dr}{dt} = \frac{S-1}{K+D} \tag{1}$$

where r is the radius of the drop and S is the saturation mixing ratio $\frac{e_s(r)}{e_s}$, which is the ratio of saturated vapor pressure over a drop to that over a flat surface. K and D describe the conduction of heat and the diffusion of the water vapor, respectively, and are given by

$$\begin{aligned} K =& \frac{L_{lv}^2 \rho_l}{\kappa R_v T^2} \\ D =& \frac{\rho_l R_v T}{e_s(T) D_v} \end{aligned}$$

where κ is molecular thermal conductivity, D_v is the water vapor diffusivity, ρ_l is the density of liquid water, L_{lv} is the latent heat of vaporization, R_v is the gas constant for vapor, T is the temperature and e_s is the saturation vapor pressure.

Supersaturation is defined as the $S-1$, where $S \equiv \frac{e_s(r)}{e_s}$. The evolution of the supersaturation is governed by two components: a source term and a sink term. First, as the temperature decreases due to adiabatic lifting, the saturation vapor pressure (e_s) decreases and therefore acts as a source for supersaturation. Conversely, as water vapor diffuses into the drop, less vapor is available in the environment and acts as a sink for supersaturation. This evolution is described by

$$\frac{dS}{dt} = a_1 u_z - a_2 \frac{dw_l}{dt}, \tag{2}$$

where u_z is the vertical velocity of the particle and w_l is the liquid water mixing ratio. The liquid water mixing ratio is a function of the droplet radius and is expressed by

$$w_l = \frac{4\pi}{3} \frac{n \rho_l r^3}{m_d} \tag{3}$$

where n is the number of droplets in the parcel, and m_d is the mass of dry air in the parcel under consideration. By applying Dalton's law of partial pressures and the Clausius-Clapeyron equation, the constants a_1 and a_2 can be found to be

$$a_1 = \frac{1}{T}\left(\frac{L_{lv} g}{R_v c_p T} - \frac{g}{R_d}\right)$$
$$a_2 = \rho_a \left(\frac{R_v T}{\epsilon e_s(T)} + \frac{\epsilon L_{lv}^2}{p T c_p}.\right),$$

where g is gravitational acceleration, c_p is the specific heat at constant pressure, R_d is the gas constant for dry air, p is pressure, and ϵ is the ratio of the mean molecular weight of water vapor to dry air.

3 Methods

Solving equations 1 and 2 is complicated because not only does $\frac{dr}{dt}$ depend on supersaturation, but $\frac{dS}{dt}$ depends on the droplet radius due to the w_l term. Thus, these equations cannot be solved analytically. Using a sufficiently small time step (0.025 s), we can calculate r and S incrementally:

$$r(i+1) = r(i) + \frac{dr(i)}{dt}\Delta t \tag{4}$$

$$S(i+1) = S(i) + \frac{dS(i)}{dt}\Delta t \tag{5}$$

where $\frac{dr}{dt}$ and $\frac{dS}{dt}$ are from equations 1 and 2, respectively.

To examine the evolution of cloud droplets for a distinct population of CCN, we consider a saturated air parcel at an initial pressure of 800 mb that is adiabatically lifted at a constant velocity of 0.1 m/s. These values of initial pressure and velocity are chosen based on Mordy (1959), who first investigated this effect. We base the initial size distribution of CCN, i.e. the initial droplet radii distribution, on that used by Mordy (1959).

The number of CCN per unit volume of air varies based on region and individual CCN size. For continental conditions, CCN particles tend to be larger and more abundant, while for maritime conditions, CCN particles are usually smaller and fewer in number density. An overall number distribution for CCN is approximated by

$$N_{CCN} = c_1(S-1)^k$$

where c_1 and k depend on the particular parcel (Curry, 1999). For general continental conditions, 1 m^3 of air would have on the order of 10^8 CCN particles (Curry, 1999). We vase our total number density off of this value, and as this does not give a distribution based on individual CCN size, we use a uniform distribution over our model's population of CCNs for simplicity.

Under the assumptions of hydrostatic balance and adiabatic cooling, temperature and pressure can be expressed as functions of altitude, and by association with vertical velocity, time:

$$T(t) = T_0 - \Gamma u_z t \tag{6}$$

$$P(t) = P_0 \left(1 - \frac{\Gamma u_z t}{T_0}\right)^{g/R_d\Gamma} \tag{7}$$

where Γ is the atmospheric lapse rate. The dry adiabatic lapse rate is an unreasonable approximation for a saturated parcel. The moist adiabatic lapse rate is a more appropriate choice, however, its value varies depending on the amount of water vapor present in the parcel. For simplicity, we chose a constant value of 4K/km for our lapse rate, which is within the range of the moist adiabatic lapse rate. The other constants and initial conditions used in our model are shown in Table 1.

4 Results and Discussion

Figure 1 shows the results of the interdependency between droplet radius and supersaturation. The results of our model indicate that the rate of increase of droplet radius is inversely related to droplet radius, that is, small droplets increase in radius faster than larger droplets. As a result, the rate of change of droplet size decreases as droplets grow, and the individual lines representing various initial droplet radii begin to converge as altitude increases. This is consistent with expectations based upon simple droplet geometry. For a large droplet with greater surface area, more mass is required to increase the droplet radius compared to a smaller droplet.

Beginning from the cloud base, supersaturation increases as expected due to cooling during adiabatic lifting. After reaching a maximum, supersaturation begins to decrease as more water condenses and $\frac{dw_l}{dt}$ becomes the dominant term in equation 2. The values for supersaturation are in agreement with observed atmospheric conditions.

Our model assumes a constant adiabatic lapse rate. In reality, the moist adiabatic lapse rate is dependent on temperature, and therefore altitude. For our purposes however, this assumption should not have significant impact. Our model also assumes a uniform distribution of several discrete sizes of CCN when in reality, the distribution would be a continuous spectrum with a varied distribution among initial radii. According to our results, all initial droplet radii were activated. Activation occurs at a unique altitude for each initial droplet radius. This altitude of activation can be seen in Figure 1, and corresponds to the point at which each line begins to deviate from the vertical. This is in disagreement with the results of Mordy (1959) in which the smallest CCN radii did not activate. The discrepancy in results may be due to the aforementioned simplification of CCN distribution.

Our model demonstrates the interdependent relationship between droplet radius and supersaturation during adiabatic lifting of an initially saturated air parcel. The assumption of hydrostatic balance and assumed vertical velocity allows both droplet radius and supersaturation to be expressed as functions of time, thus the relationships can be solved using incremental time steps. The results of the model confirm expected relationships between radius increase and supersaturation dependence with altitude.

Constant or Initial Condition	Symbol	Value	Units
Enthalpy of Vaporization for Water	$L_l v$	$2.5e^6$	J/kg
Density of Liquid Water	ρ_l	1000	kg/m^3
Water Vapor Gas Constant	R_v	461	$J/(Kkg)$
Thermal Conductivity for $273K$	κ	$2.40e^{-2}$	$J/(msK)$
Water Vapor Diffusivity at $1000mb$ at $273K$	D_v	$2.21e^{-5}$	m^2/s
Adiabatic Lapse Rate	Γ	.004	K/m
Gravitational Constant	g	9.81	m/s^2
Dry Gas Constant	R_d	287	$J/(kgK)$
Heat Capacity at Constant Pressure for Dry Air	c_p	1004	$J/(kgK)$
Ratio of Molar Masses of Water Vapor to Dry Air	ϵ	0.622	
Density of Air	ρ_a	1.29	kg/m^3
Reference Saturation Vapor Pressure at $273K$	e_{s0}	610.70	Pa
Reference Temperature for CC Equation	T_0	273	K
Initial Temperature	T_i	283	K
Mass of Dry Air	m_d	0.6	kg
Initial Supersaturation	S_0	1.00	$\%$
Vertical Velocity	u_z	0.1	m/s
Initial Ambient Pressure	P_0	$8e^5$	Pa

Table 1. List of constants and initial conditions used in the model.

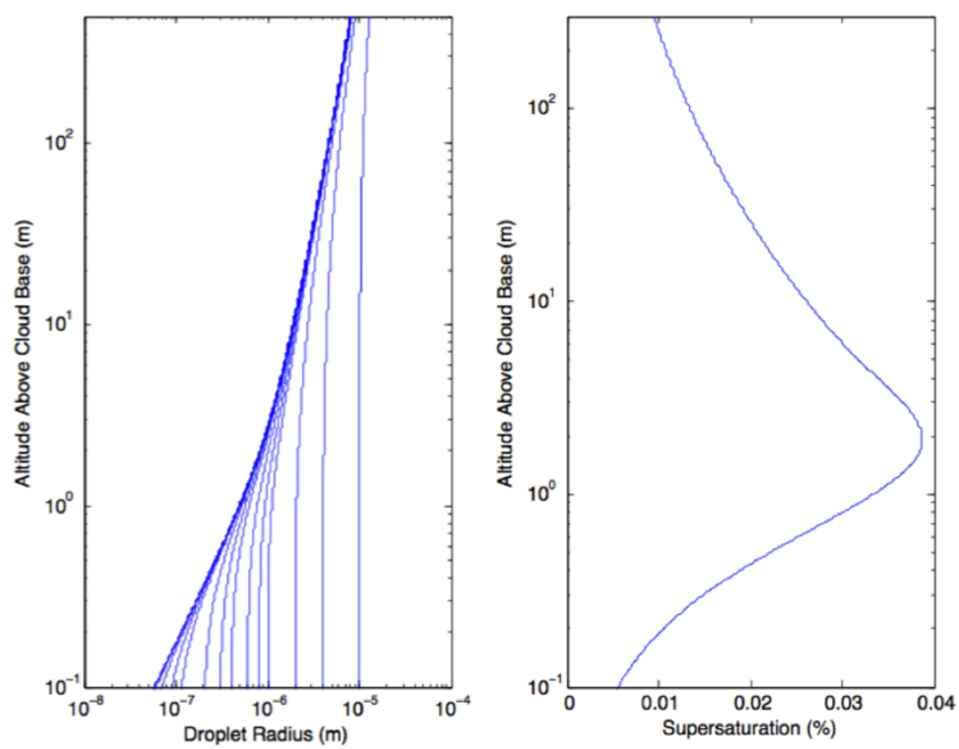

Figure 1. Evolution of a population of droplets with altitude (left).
Evolution of supersaturation of an air parcel with altitude (right).

5 References

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